Version A

Midterm Exam – Econ 2450
October 8, 2025
Department of Economics, York University

Instructions: For some problems you need to show your calculations; for others you should only write the correct answer. Follow the instructions in italics for each problem. Answers must be given on the answer sheets provided. Ask for extra answer sheets if needed. Do not fold the answer sheets or write on the back.

Problem 1. Labor Supply [4 marks]

Consider a model with $\overline{L}$ workers. If they do not work they earn b in unemployment benefits, and if they do work they earn a wage w. Workers maximize their earnings, implying that total labor supply, L^S, is given by:

$$L^S = \begin{cases} \overline{L} & \text{if } w \geq b, \\ 0 & \text{if } w < b. \end{cases} \tag{1}$$

For all of problems (a)-(d) below, you should not show how you arrived at your answer, only draw the graph or write the expression or word correctly.

(a) Illustrate the supply function in (1) in a diagram with w on the vertical axis and L^S on the horizontal axis. Indicate $\overline{L}$ and b on suitable axes. [1 mark]

(b) Now instead assume that half of the $\overline{L}$ workers value leisure so much that they are willing to work only if the wage is at least *twice* as large as the benefit, $w \geq 2b$. The other half is willing to work as long as $w \geq b$. No one is willing to work if $w < b$. Write an equation for L^S corresponding to that in (1). [1 mark]

(c) Illustrate your answer to (b) in a diagram with w on the vertical axis and L^S on the horizontal axis. Indicate $\overline{L}$, $\overline{L}/2$, b, and $2b$ on suitable axes. [1 mark]

(d) What does D in DSGE model stand for? [1 mark]

Problem 2. Labor Demand [4 marks]

Consider a model where there are two types of firms producing the same good. One type of firm is more productive. We call the high-productive firms type A and low-productive firms type B. There are N firms in total and δN of these are of type A (more productive), where $0 < \delta < 1$, and $(1-\delta)N$ firms are type B (less productive). Output of each high-productive firm equals $A\left(L_A\right)^{\gamma}$, where $A > 0$ and $\gamma \in (0,1)$ are exogenous parameters, and L_A is the number of workers hired by each such firm. Output of each type-B (low-productive) firm equals $\theta A\left(L_B\right)^{\gamma}$, where $0 < \theta < 1$, and L_B is the number of workers hired by each such firm. The parameters A and γ are the same across firms, and all firms face the same market wage, w.

Type-A firms choose L_A to maximize their profits, given by $\pi_A = A\left(L_A\right)^{\gamma} - wL_A$. This gives demand for labor from each such firm as follows:

$$L_A = \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}.$$

For (a)-(b) below you should show how you arrived at your answer. For (c) you should not, just draw the diagram correctly.

(a) Find the profit-maximizing level of L_B for firms of type B, corresponding to the expression for L_A above. Show each step. [1 mark]

(b) Find an expression for aggregate demand for labor, L^D, which equals the sum of demand across all firms. For full mark, your answer should be written as the product of two factors: one involving only θ, δ, and γ, and another involving only A, N, γ, and w. Show each step. (Hint: in class we studied a model with N_A and N_B firms having productivity parameters A and B, respectively; the setting here corresponds to one where $N_A = \delta N$, $N_B = (1-\delta)N$, and $B = \theta A$.) [2 marks]

(c) Illustrate aggregate labor demand, L^D, in a diagram with w on the vertical axis and L^D on the horizontal axis. Show how L^D shifts in response to an increase in δ. [1 mark]

Problem 3. Search and Matching [4 marks]

Consider a search-and-matching model. The number of matches (M) equals

$$M = \frac{QV}{Q+V},$$

where Q is the number of agents searching for employment, and V is the number of vacancies. The probability of a firm filling a vacancy equals $p_f = M/V$, and the probability of a worker finding a job equals $p_w = M/Q$.

The firm's profit per vacancy equals $p_f\left(A - w\right) - k$, where A is a productivity parameter, w the wage rate, and k the cost of posting the vacancy. The worker's unemployment benefit is denoted b, and the wage, w, is set such that $w - b = \beta(A - b)$, where $\beta \in (0,1)$ denotes the worker's bargaining power. We let $\theta = V/Q$ denote job-market tightness (i.e., vacancies per job searcher).

For problems (a)-(c) below you should not show how you arrived at your answer, only write the expression correctly. For (d) you should show all your calculations.

(a) Write an expression for p_f in terms of θ. [0.5 marks]

(b) Write an expression for p_w in terms of θ. [0.5 marks]

(c) Find an expression for the firm's surplus, $A - w$, in terms of β, b, and A. [1 mark]

(d) The unemployment rate equals probability that the worker does not find a job, $u = 1 - p_w$. Use the firm's zero-profit condition, and other information provided, to find an expression for u in terms of k, A, b, and β. Show each step. (We assume that the exogenous variables are such that $0 < u < 1$.) [2 marks]

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Version B

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(b) Write an expression for p_w in terms of θ. [0.5 marks]

(c) Find an expression for the firm's surplus, $A - w$, in terms of β, b, and A. [1 mark]

(d) Use the firm's zero-profit condition, and other information provided, to find an expression for equilibrium θ. Your answer should be in terms of k, A, b, and β. Show each step. [2 marks]

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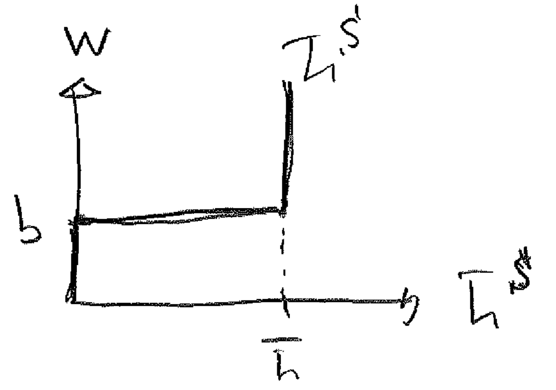
VA 2450 solutions Oct. 8 midterm

1 (a)

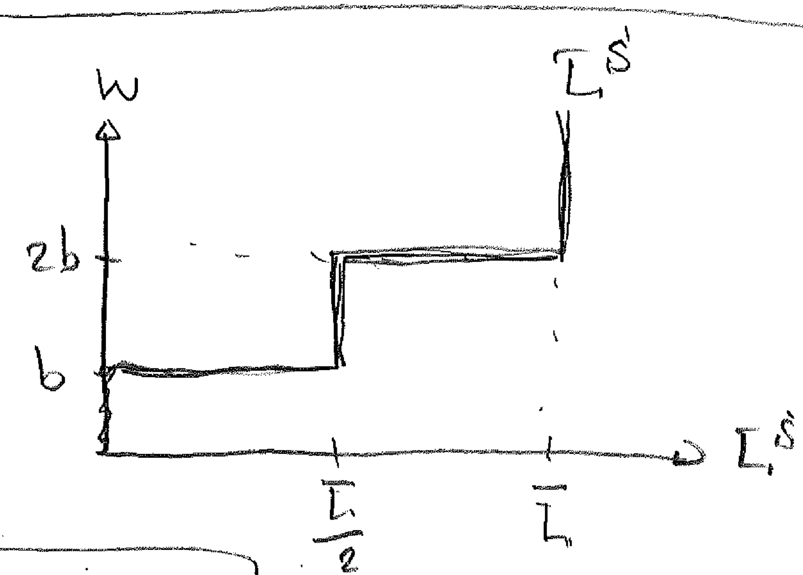


(b)

$$L_1^S = \begin{cases} \bar{L} & \text{if } w \geq 2b \\ \bar{L}/2 & \text{if } b \leq w < 2b \\ 0 & \text{if } w < b \end{cases}$$

Ignore whether weak/strict inequalities are correct (not needed)

(c)



(d) Dynamic

2 (a)

$$\boxed{\pi_B = \theta A (L_B)^{\gamma} - w L_B}$$

FOC:

$$\boxed{\frac{\partial \pi_B}{\partial L_B} = \gamma \theta A (L_B)^{\gamma - 1} - w = 0}$$

needed

Solve for L_B:

$$\left.\begin{aligned} \gamma \theta A (L_B)^{\gamma-1} &= w \\ L_B^{\gamma-1} &= \frac{w}{\gamma \theta A} \\ L_B^{1-\gamma} &= \frac{\gamma \theta A}{w} \end{aligned}\right\}$$

These steps not needed

$$\boxed{L_B = \left(\frac{\gamma \theta A}{w}\right)^{\frac{1}{1-\gamma}}}$$

Final answer (needed)

2 (b)

$$L^D = \underbrace{\delta N \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}}_{\substack{\text{Demand from} \\ \text{high prod. (type A)} \\ \text{firms}}} + \underbrace{(1-\delta) N \left(\frac{\theta \gamma A}{w}\right)^{\frac{1}{1-\gamma}}}_{\substack{\text{Demand} \\ \text{from low-prod.} \\ \text{(type B) firms}}}$$

$$\boxed{L^D = \left[\delta + (1-\delta)\theta^{\frac{1}{1-\gamma}}\right] N \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}}$$

Answer

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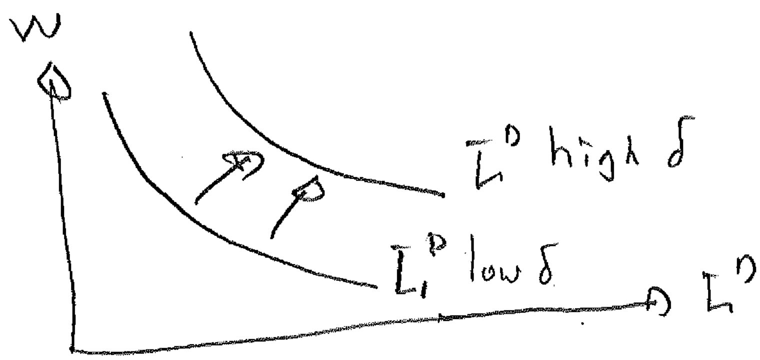
2 (c)

$$\delta \uparrow \Rightarrow \left[\delta + (1-\delta)\theta^{\frac{1}{1-\gamma}}\right] \uparrow \Rightarrow L^D \uparrow$$

(since $\theta < 1$)

Not needed



sufficient answer

3(a)

not needed

$$\boxed{P_f = \frac{M}{V} = \frac{Q}{Q+V} = \frac{1}{1+\frac{V}{Q}} = \frac{1}{1+\theta}}$$

$$\boxed{P_f = \frac{1}{1+\theta}}$$ ⟸ sufficient answer

(b) $$\boxed{P_w = \frac{M}{Q} = \frac{V}{Q+V} = \frac{\frac{V}{Q}}{1+\frac{V}{Q}} = \frac{\theta}{1+\theta}}$$

not needed

$$\boxed{P_w = \frac{\theta}{1+\theta}}$$ ⟸ sufficient

(c) $A - w = A - [b + \beta(A-b)]$ } not needed

$$\boxed{\begin{aligned} A - w &= A - b - \beta(A-b) \\ &= (1-\beta)(A-b) \end{aligned}}$$ ⟸ either is okay

3(d) * Many solutions possible

* Here the one I think is easiest

First note that

$$u = 1 - p_w = 1 - \left(\frac{\theta}{1+\theta}\right) = \frac{1}{1+\theta} = p_f$$

So $u = p_f$

Next:

Zero profit $\Rightarrow$ $p_f(A - w) = k$

From (c): $A - w = (1-\beta)(A-b)$

$$u = p_f = \frac{k}{A - w} = \frac{k}{(1-\beta)(A-b)}$$

$$\boxed{u = \frac{k}{(1-\beta)(A-b)}}$$ $\Leftarrow$ Final Answer

VB

24JO Solutions Oct. 8 midterm

1(a)-(c) Same as VA

1(d) Stochastic

2 Same as VA, except
δ = share low-prod. firms
and θ replaced by β

2(a) $\boxed{\begin{aligned}&\pi_B = \beta A(L_{1B})^{\gamma} - wL_{1B}\\ &\text{FOC:}\\ &\frac{\partial \pi_B}{\partial L_{1B}} = \gamma\beta A(L_{1B})^{\gamma-1} - w = 0\end{aligned}}$

Needed

2(a) cont'd

Solve FOC for L_B:

$$\gamma \beta A L_B^{\gamma-1} = w$$

$$L_B^{\gamma-1} = \frac{w}{\gamma\beta A}$$

These steps not needed

$$\boxed{L_B = \left(\frac{\gamma\beta A}{w}\right)^{\frac{1}{1-\gamma}}}$$

Final answer (needed)

(b)

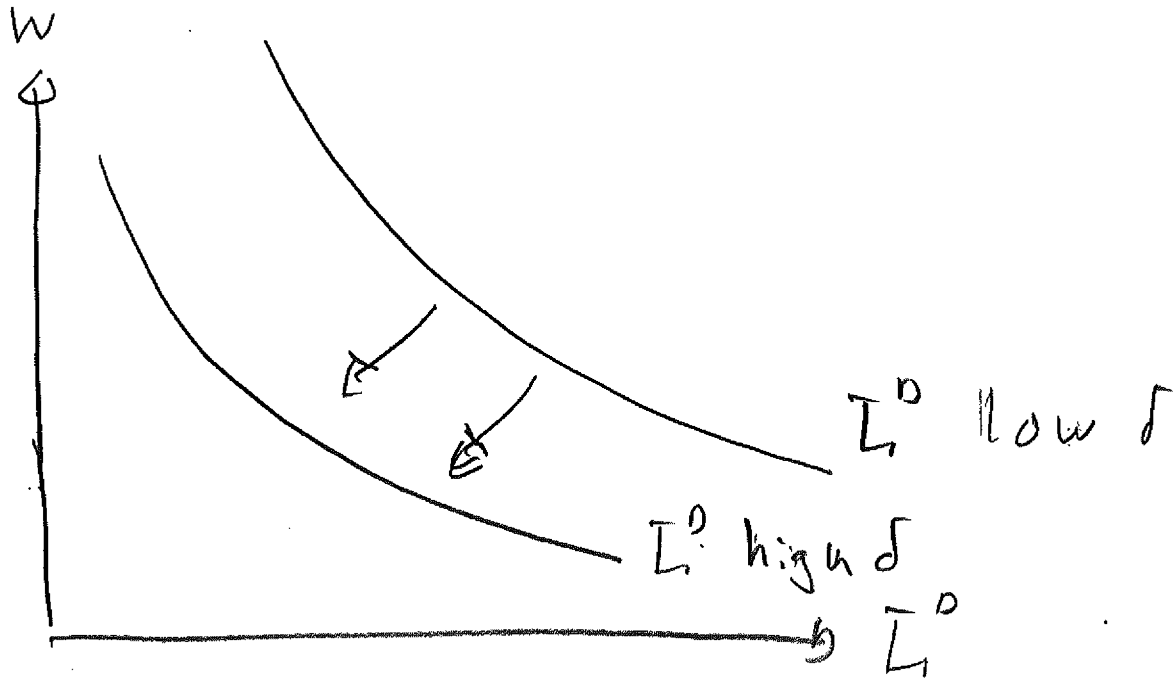
$$L^D = \underbrace{(1-\delta) N \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}}_{\text{demand from high prod. firms}} + \underbrace{\delta N \left(\frac{\beta\gamma A}{w}\right)^{\frac{1}{1-\gamma}}}_{\text{demand from low prod. firms}}$$

$$\boxed{L^D = \left[1-\delta + \delta\, \beta^{\frac{1}{1-\gamma}}\right] N \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}}$$

not needed

(c) $\delta \uparrow \Rightarrow \left[1-\delta+\delta\beta^{\frac{1}{1-\gamma}}\right] \downarrow$

(since $\beta < 1$) $\Rightarrow \bar{L}_i^D \downarrow$



sufficient answer

3 (a) – (c) same as V.A

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3 (d) Find equil. θ

From (a): $P_f = \dfrac{1}{1+\theta}$

From (c): $A - w = (1-\beta)(A-b)$

zero profits $\Rightarrow$ $P_f = \dfrac{k}{A-w}$

$$\Rightarrow \quad \frac{1}{1+\theta} = \frac{k}{(1-\beta)(A-b)}$$

Solve for θ:

$$1+\theta = \frac{(1-\beta)(A+b)}{k}$$

Either expression is okay

$$\boxed{\theta = \frac{(1-\beta)(A+b)}{k} - 1 = \frac{(1-\beta)(A-b) - k}{k}}$$