Some slides for Econ 2450
Continuously updated

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Introduction

Some things that one generally needs to know to do well in macroeconomics

- Theory: using, understanding economic models

- Programming: writing computer code for numerical simulations

- Data: finding data sources (e.g. the National Accounts); statistics, econometrics

In this course: focus on 1st point

Does *not* mean 2nd and 3rd points are not important; dealt with in other courses

Important concept: endogenous and exogenous variables

A variable that is endogenous is explained *within the model*

A variable that is exogenous comes from *outside the model*

Exogenous variables are often called (exogenous) *parameters*

Note: a variable that is endogenous in one model can be exogenous in another, and vice versa

Typical exogenous variables are those that describe utility functions (e.g. time preferences) and production functions (e.g. productivity)

Typical endogenous variables are prices and quantities determined in the market place

A The IS-LM model and its cousins

The Keynesian multiplier

The National Accounts identity:

$$Y = C + I + G + NX$$

Notation:

- Y denotes output, or income (GDP)
- $C = C(Y - T)$ denotes household consumption
- $C(Y - T)$ called consumption function
  - Assumption: $0 < C'(Y - T) < 1$

- $C'(Y - T)$ called *marginal propensity to consume*

- T denotes taxes, exogenous

- I denotes investment, exogenous

- G denotes government consumption, exogenous

- NX denotes net exports
  - Here: closed economy, so $NX = 0$

Idea in "Keynesian" economics:

- Increase one of the demand components on the right-hand side in NA equation leads to increase in Y

- Economy said to be "demand driven"

- Plausible (argue Keynesians) if there are many idle resources (unemployed workers, unused capital)

- Might suggest role for government to change G, called *fiscal policy*

Examine effect on Y due to change in G

Think of Y as function of G, differentiate with respect to G

$$\frac{dY}{dG} = C'(Y - T)\frac{dY}{dG} + 0 + 1$$

Solve for $\frac{dY}{dG}$

$$\frac{dY}{dG} = \frac{1}{1 - C'(Y - T)}$$

- Note $\frac{1}{1-C'(Y-T)} > 1$, called the *Keynesian multiplier*

- Means that a \$1 increase in G leads to *more* than a \$1 increase in Y, a *multiplier effect*

- Suggested remedy to get economy out of recession (boost Y)

Illustrate in diagram with Y and $C + I + G$ on vertical axis, and Y on horizontal axis

Called the *Keynesian Cross*

How big is multiplier?

- Examine US data over (changes in) Y and G
- Problem:
  - Want causality to run from G to Y
  - Could run other way (from Y to G), e.g. through tax revenues
- Solution: look at war-induced defense spending (G up due to war)
- Robert Barro (WSJ January 2009): multiplier likely *less* than one

How about budget deficit?

- At given T, increase in G brings increase in government's budget deficit (or reduction in surplus)

- Deficit can affect interest rates and thus investment (see IS-LM model later)

- If financed through borrowing, lower consumption in future when paying off loan

The budget deficit

Commonly heard argument:

- Increase in G raises Y, which raises tax revenues (T)
- Fiscal expansion might "pay for itself"
- Can it? Let's take the model seriously.

More notation:

- Let C be given by an *explicit functional form*

$$C = C(Y - T) = a + b(Y - T)$$

- b same as $C'(Y - T)$, $b \in (0, 1)$

Constant tax rate:

- Let T be given by $T = \tau Y$, where $\tau \in (0, 1)$

Together:

$$C = a + b(1 - \tau)Y$$

Illustrate in diagram with C on vertical axis and Y on horizontal

Use NA identity:

$$Y = C + I + G = \underbrace{a + b(1 - \tau)Y}_{C} + I + G$$

Solve for Y

$$Y = \frac{a + I + G}{1 - b(1 - \tau)}$$

Define budget deficit as $D = G - T$

Does D increase or decrease when we raise G? By how much? Note: T is endogenous!

$$
\begin{aligned}
D &= G - T \\
&= G - \tau Y \\
&= G - \tau \left[\frac{a + I + G}{1 - b(1-\tau)} \right] \\
&= G \left[1 - \frac{\tau}{1 - b(1-\tau)} \right] - \frac{\tau (a + I)}{1 - b(1-\tau)} \\
&= G \left[\frac{1 - b(1-\tau) - \tau}{1 - b(1-\tau)} \right] - \frac{\tau (a + I)}{1 - b(1-\tau)} \\
&= G \underbrace{\left[\frac{(1-b)(1-\tau)}{1 - b(1-\tau)} \right]}_{\in(0,1)} - \frac{\tau (a + I)}{1 - b(1-\tau)},
\end{aligned}
$$

Fiscal expansion pays for itself partly ($dD/dG < 1$) but not fully ($dD/dG > 0$)

Illustrate in diagram with D in vertical axis and G on horizontal axis

The IS-LM model

News:

- r denotes the interest rate
- I still denotes investment, where $I = I(r)$
- $I(r)$ called investment function
  - Assumption: $I'(r) < 0$; higher r means it is more expensive to borrow to finance investments (or opportunity cost is higher)
- T and G exogenous

The IS Curve

The IS curve is defined as combinations of r and Y such that

$$Y = C(Y - T) + I(r) + G$$

Draw in diagram with r on vertical axis, Y on horizontal axis

Slope is negative

To see this check that $\frac{dr}{dY} < 0$ along the IS curve

Intuition: when r is high, then I is low, which means low Y

The LM curve

Notation

- M denotes nominal money supply ($)

- P denotes general price level

- $L(r, Y)$ denotes demand for real money balances (think of as the real value of the cash that shoppers hold in their pockets)

  - Assumption: $\frac{\partial L(r,Y)}{\partial r} < 0$; when r increases agents prefer to hold more money in the bank (where it earns interest) and less in their pockets

  - Assumption: $\frac{\partial L(r,Y)}{\partial Y} > 0$; when Y increases agents want to consume more, and need to hold more cash

The LM curve is defined as combinations of r and Y such that

$$\frac{M}{P} = L(r, Y)$$

Draw in diagram with r on vertical axis, Y on horizontal axis

Slope is positive

To see this check that $\frac{dr}{dY} > 0$ along the LM curve

The Mundell-Fleming (IS*-LM*) model

Describes small, open economy

Similar to IS-LM; some news:

Interest rate exogenous, denoted r^* (world interest rate)

Thus, $I^* = I(r^*)$ effectively exogenous

New variables, notation:

- $NX = NX(e)$ denotes net exports (exports minus imports)

- e denotes the exchange rate = units of foreign currency (say ¥) per unit of domestic currency ($); a high e means domestic currency is expensive

- Assumption: $NX'(e) < 0$; more expensive $ (and cheaper ¥) means less demand for Canadian goods in Japan, higher demand for Japanese goods in Canada

The IS* curve

The IS* curve is defined as combinations of e and Y such that

$$Y = C(Y - T) + I^* + G + NX(e)$$

Draw in diagram with e on vertical axis, Y on horizontal axis

Slope is negative

To see this check that $\frac{de}{dY} < 0$ along the IS* curve

The LM* curve

Notation

- λ denotes spending share on domestic goods; $1 - \lambda$ on foreign goods

- P_d denotes price of domestic goods (in $); P_f foreign goods (in ¥)

General price level in home country (Canada) becomes

$$P = \lambda P_d + (1 - \lambda)\frac{P_f}{e}$$

Note that the $ price of imported goods is P_f/e; see problem set

The LM* curve is defined as combinations of e and Y such that

$$\frac{M}{\lambda P_d + (1-\lambda)\frac{P_f}{e}} = L(r^*, Y)$$

(Recall that r^* is exogenous)

Draw in diagram with e on vertical axis, Y on horizontal axis

Slope is positive

To see this check that $\frac{de}{dY} > 0$ along the LM* curve

Discussion

- Keynesian models are old
  - J.M. Keynes' ideas formulated in "General Theory of Employment, Interest, and Money" published 1936
  - IS-LM model developed by J. Hicks in article in *Econometrica* in 1937
- Not used any longer among professional academic economists. Why?

- Main shortcoming: no *micro foundations*
  - No utility maximization by agents; consumption and real money demand functions not derived
  - No profit maximization by firms: investment function, or production/pricing decisions not derived
  - Static (single-period) model, but intertemporal (multi-period) concepts: saving, interest rates
  - No government budget constraint; T, G, and M not explicitly connected

What type of models do (acedemic) economists use instead?

Common tool: *Dynamic Stochastic General Equilibrium (DSGE) models*.

- *Dynamic* means it has many time periods
  - enables the model to explain saving and investment decisons (decisions made over time)

- *Stochastic* means random
  - enables the model to explain how agents behave in an uncertain environment; capture concepts like risk

- *General Equilibrium* means that several markets are in equilibrium simultaneously

  – enables model to deal with the effects on the whole economy (as opposed to e.g. just one sector)

- *Models* are simplified, imagined economies

  – Models are "purposeful simplifications that serve as guides to the real world, they are not the real world." (VV Chari)

Rest of this course about introducing various sorts of models

- Not (full-scale) DSGE models, but sharing some components

- Meant to get you started, wet your appetite

B Labor markets

Interesting macroeconomic phenomenon: unemployment

- Higher is some countries, and some regions of some countries, than others

- Rises during recessions, falls in expansions

Understanding why requires that we model labor markets explicitly

Model demand for labor from firms, supply of labor from households

Perfect Competition

Perfect Competition (PC) here means that there are many firms, many workers: each takes the price of labor (the real wage rate) as given

Later: workers acting as monopolist (single supplier); Union Wage Setting

Other models (not discussed further here): one firm being monopsonist (single buyer)

Notation:

- N denotes the number of firms; assume N is "large"
- $\overline{L}$ denotes total labor force; roughly same as total number of workers; here $\overline{L}$ is exogenous; assume $\overline{L}$ is "large"
- w denotes (real) wage
- b denotes a government benefit that a worker gets if choosing not to work
- L^S denotes supply of labor; function of w
- L^D denotes demand for labor; function of w

Supply of labor

Assume the following:

- All $\overline{L}$ want to work if wage higher than (or equal to) the benefit ($w \geq b$)

- No one wants to work if wage lower than the benefit ($w < b$)

$$L^S = \begin{cases} \overline{L} & \text{if } w \geq b, \\ 0 & \text{if } w < b. \end{cases}$$

Draw in diagram with w on vertical axis, L^S on horizontal axis

(Similar shape when derived from utility maximization problem with log utility, but then w must exceed a threshold *higher* than b for workers to want to work)

Note:

- Agents here either work full time, or not at all

- Model thus captures to what extent agents work at all (*extensive margin*), rather than how *much* they work (*intensive margin*)

Demand for labor

More notation:

- Output of firm who hires L workers equals AL^{γ}
- $A > 0$ is a productivity parameter
- $0 < \gamma < 1$ measures the elasticity of output with respect to L
- π is the firm's profit

This gives

$$\pi = AL^{\gamma} - wL$$

(We normalize the price of the firms' output to 1; w is the *real* wage and π the *real* profit)

Each firm chooses L to maximize π, taking w as given

First-order condition:

$$\frac{\partial \pi}{\partial L} = 0$$

gives each firm's optimal L as

$$L = \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}$$

Since each of the N firms hires L workers, aggregate labor demand, L^D, equals NL

$$L^D = N\left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}}$$

(See problem set for details)

Draw in diagram with w on vertical axis, L^D on horizontal

(May help to invert to find shape)

Labor market equilibrium

Illustrate in labor market diagram

Two types of equilibrium:

(A) Either $w = b$ and $L^D \leq L^S = \overline{L}$; or

(B) $w > b$ and $L^S = L^D = \overline{L}$

Under (A) some who are willing to work at the prevailing wage ($w = b$) may not find a job

But: (almost) indifferent; unemployment in that sense *voluntary*

Define

$$\widehat{A} = \frac{b}{\gamma}\left(\frac{\overline{L}}{N}\right)^{1-\gamma}$$

Equilibrium w is given by

$$w = \begin{cases} b & \text{if } A < \widehat{A} \\ \gamma A \left(\frac{N}{\overline{L}}\right)^{1-\gamma} & \text{if } A \geq \widehat{A} \end{cases}$$

Note that $A \geq \widehat{A}$ implies that

$$w = \gamma A \left(\frac{N}{\overline{L}}\right)^{1-\gamma} \geq b$$

Interpret in labor market diagram:

- If $A < \widehat{A}$, then L^D intersects L^S *below* the "kink" (less than full employment)

- If $A > \widehat{A}$, then L^D intersects L^S *above* the "kink" (full employment)

- If $A = \widehat{A}$, then L^D intersects L^S *exactly at* the "kink" (full employment)

How about unemployment?

More notation:

- u denotes the unemployment rate = fraction of the $\overline{L}$ workers who do *not* work

$$u = \frac{\overline{L} - L^D}{\overline{L}} = 1 - \frac{L^D}{\overline{L}}$$

Now we see the following:

- When $A \geq \widehat{A}$, it holds that $w \geq b$ and $L^S = L^D = \overline{L}$, so $u = 0$

- When $A < \widehat{A}$, it holds that $w = b$; setting $w = b$ in expression for L^D gives

$$L^D = N \left(\frac{\gamma A}{b} \right)^{\frac{1}{1-\gamma}}$$

Equilibrium u is thus given by

$$u = \begin{cases} 1 - \frac{N}{\overline{L}} \left(\frac{\gamma A}{b}\right)^{\frac{1}{1-\gamma}} & \text{if } A < \widehat{A} \\ \\ 0 & \text{if } A \geq \widehat{A} \end{cases}$$

Illustrate:

- In figure with u on vertical axis, A on horizontal

- In figure with w on vertical axis, A on horizontal

What happens to u and w as A increases? Think of rise in A as economic expansion; fall in A as recession

Union Wage Setting

Under Perfect Competition (PC) unemployed workers have same income as employed

- Implies unemployment voluntary (strange?)
- Inconsistent with the data: premium to having a job
- Model cannot explain why regions with high rates of union membership have higher unemployment rates

Now instead let a union set wages

Firms still hire workers to maximize profits

Union maximizes the expected wage of a worker

Notation similar to PC model; some news:

- E denotes the expected wage
- u still denotes unemployment rate, also probability not finding a job; $1-u$ is probability finding job

Thus:

$$
\begin{aligned}
E &= \underbrace{u}_{\text{prob. not finding job}} \underbrace{b}_{\text{unempl. benefit}} + \underbrace{1-u}_{\text{prob. finding job}} \underbrace{w}_{\text{wage}} \\
&= b + (1-u)(w-b)
\end{aligned}
$$

Idea: union faces trade-off; setting higher w implies lower probability finding job

Task:

- First find E as function of w
- Then find w that maximizes E

Finding E as function of w

Recall that $E = b + (1 - u)(w - b)$

Need to find u as function of w

Start off with

$$u = \frac{\overline{L} - L^D}{\overline{L}} = 1 - \frac{L^D}{\overline{L}}$$

but note that $u = 0$ when $L^D \geq \overline{L}$

From firms' profit-maximization we derived demand for labor as function of w

$$L^D = N \left(\frac{\gamma A}{w} \right)^{\frac{1}{1-\gamma}}$$

We now see that $u = 0$ when w set so low that

$$L^D = N \left(\frac{\gamma A}{w} \right)^{\frac{1}{1-\gamma}} \geq \overline{L}$$

That is, $u = 0$ when

$$w \leq \widehat{w} = \gamma A \left(\frac{N}{\overline{L}} \right)^{1-\gamma}$$

And if $w > \widehat{w}$, then

$$u = 1 - \frac{L^D}{\overline{L}} = 1 - \frac{N}{\overline{L}} \left(\frac{\gamma A}{w} \right)^{\frac{1}{1-\gamma}} \in (0, 1)$$

Graphical interpretation: $\widehat{w}$ given by intersection between L^D and $\overline{L}$

This gives u as function of w:

$$u = u(w) = \begin{cases} 1 - \frac{N}{\overline{L}} \left(\frac{\gamma A}{w} \right)^{\frac{1}{1-\gamma}} & \text{if } w > \widehat{w} \\ 0 & \text{if } w \leq \widehat{w} \end{cases}$$

Illustrate in diagram with w on horizontal axis, and u on vertical

Substituting into expression for E gives E as function of w

$$E(w) = b + [1 - u(w)]\,(w - b)$$

This gives:

$$E = E(w) = \begin{cases} b + \frac{N}{\overline{L}} \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}} (w - b) & \text{if } w > \widehat{w} \\ w & \text{if } w \leq \widehat{w} \end{cases}$$

Illustrate in diagram with w on horizontal axis, and E on vertical

Finding w that maximizes E

Let $E(w)$ when $w > \widehat{w}$ be denoted as $G(w)$

$$
\begin{aligned}
G(w) &= b + \frac{N}{\overline{L}} \left(\frac{\gamma A}{w} \right)^{\frac{1}{1-\gamma}} (w - b) \\
&= b + \frac{N}{\overline{L}} (\gamma A)^{\frac{1}{1-\gamma}} \left\{ w^{-\frac{1}{1-\gamma}} (w - b) \right\}
\end{aligned}
$$

Draw figure to understand shape of $E(w)$

Note that $E(w)$ is increasing linearly for $w \leq \widehat{w}$; thus the max point of $E(w)$ cannot be to the left of $\widehat{w}$; must be at, or to the right of, $\widehat{w}$

Two cases:

- If $G(w)$ is maximized to the left of $\widehat{w}$, then $E(w)$ is maximized at $w = \widehat{w}$ (optimal for union to choose full employment)

- If $G(w)$ is maximized to the right of $\widehat{w}$, then $E(w)$ is maximized where $G(w)$ is maximized (optimal for union to choose some unemployment)

Find w that maximizes $G(w)$

First-order condition:

$$
\begin{aligned}
& G'(w) \\
= \; & \frac{N}{\overline{\overline{L}}}(\gamma A)^{\frac{1}{1-\gamma}}\left[\left(-\frac{1}{1-\gamma}\right)w^{-\frac{1}{1-\gamma}-1}(w-b)+w^{-\frac{1}{1-\gamma}}\right] \\
= \; & \frac{N}{\overline{\overline{L}}}(\gamma A)^{\frac{1}{1-\gamma}}\,w^{-\frac{1}{1-\gamma}}\left[\left(-\frac{1}{1-\gamma}\right)w^{-1}(w-b)+1\right] \\
= \; & -\frac{N}{\overline{\overline{L}}}(\gamma A)^{\frac{1}{1-\gamma}}\,w^{-\frac{1}{1-\gamma}}\underbrace{\left[\left(\frac{1}{1-\gamma}\right)\frac{w-b}{w}-1\right]}_{=0}=0
\end{aligned}
$$

Solving for w gives

$$w = \frac{b}{\gamma}$$

Is b/γ greater or less than $\widehat{w}$?

Recall:

$$\widehat{w} = \gamma A \left(\frac{N}{\overline{\overline{L}}}\right)^{1-\gamma}$$

So $b/\gamma \geq \widehat{w}$, when

$$\frac{b}{\gamma} \geq \gamma A \left(\frac{N}{\overline{\overline{L}}}\right)^{1-\gamma}$$

Rewrite condition in terms of A:

$$A \leq \frac{b}{\gamma^2} \left(\frac{\overline{L}}{N}\right)^{1-\gamma} = \frac{\widehat{A}}{\gamma},$$

where we borrow notation from PC model:

$$\widehat{A} = \frac{b}{\gamma}\left(\frac{\overline{L}}{N}\right)^{1-\gamma}$$

Thus the w that maximizes $E(w)$ is given by:

$$w = \begin{cases} \widehat{w} = \gamma A \left(\frac{N}{\overline{L}}\right)^{1-\gamma} & \text{if } A > \frac{\widehat{A}}{\gamma} \\ \\ \frac{b}{\gamma} & \text{if } A \leq \frac{\widehat{A}}{\gamma} \end{cases}$$

How about unemployment?

- Recall that there is full employment when $w = \widehat{w}$, i.e., when $A > \widehat{A}/\gamma$

- Vice versa, when $A \leq \widehat{A}/\gamma$, the rate of unemployment is given by setting $w = b/\gamma$ in expression for $u(w)$

Expression for u when union sets w to maximize $E(w)$ becomes:

$$u = \begin{cases} 0 & \text{if } A > \frac{\widehat{A}}{\gamma} \\ u(\frac{b}{\gamma}) = 1 - \frac{N}{\overline{L}} \left(\frac{\gamma^2 A}{b} \right)^{\frac{1}{1-\gamma}} & \text{if } A \leq \frac{\widehat{A}}{\gamma} \end{cases}$$

Illustrate in diagram with A on horizontal axis

Discussion

Two models, PC and UWS

Comparison of outcomes

- Unemployment voluntary under PC, not under UWS

- Unemployment and wages higher under UWS than PC for low enough A; outcomes same for high enough A

- Differences in employment fluctuations over the business cycle

  * Let A vary over the interval $[\widehat{A}, \frac{\widehat{A}}{\gamma}]$

  * Under UWS: no effect on wages, only on unemployment

* Under PC: no effect on unemployment (since $u = 0$), only on wages

* UWS model can explain *wage rigidity*

UWS model presented here: all workers have same probability getting a job

Richer UWS models: *insider-outsider* models

- Wages set by senior workers, facing lower risk of unemployment (insiders)

- Union then tends to set higher wages, higher unemployment for outsiders

Efficiency Wages

Idea: the better workers are paid, the more productive they are

Ideally: should be derived from worker's utility max. problem (more advanced courses)

Here: just postulate that A depends on w

$$A = \begin{cases} \overline{A} & \text{if } w \geq \overline{w} \\ 0 & \text{if } w < \overline{w} \end{cases}$$

where $\overline{w} > 0$ and $\overline{A} > 0$ are exogenous

In words: workers are productive only if paid at least $\overline{w}$

Firm profits:

$$\pi = \begin{cases} \overline{A}L^{\gamma} - wL & \text{if } w \geq \overline{w} \\ -wL & \text{if } w < \overline{w} \end{cases}$$

To derive optimal L, note this:

- Never optimal to pay workers less than $\overline{w}$; thus, if $w < \overline{w}$, then no worker is hired ($L = 0$)

- If $w \geq \overline{w}$, same profit max. problem as before, but with $\overline{A}$ replacing A

Thus:

$$L^D = \begin{cases} N\left(\frac{\gamma\overline{A}}{w}\right)^{\frac{1}{1-\gamma}} & \text{if } w \geq \overline{w} \\ 0 & \text{if } w < \overline{w} \end{cases}$$

L^S same as before

Draw in diagram with w on vertical axis, L^D and L^S on horizontal

Trivial equilibrium always exists: L^D and L^S both zero, $w < \overline{w}$ and $w < b$

But assume the following:

$$\overline{w} > b$$

$$\overline{L} > N \left(\frac{\gamma \overline{A}}{\overline{w}} \right)^{\frac{1}{1-\gamma}}$$

Then there also exists an equilibrium with less than full employment ($L^D < \overline{L}$), and $w = \overline{w}$

Why?

- All $\overline{L}$ workers willing to work if $w = \overline{w} > b$, so $L^S = \overline{L}$

- $\overline{L} > N \left(\frac{\gamma \overline{A}}{\overline{w}} \right)^{\frac{1}{1-\gamma}}$ ensures labor demand falls short of $\overline{L}$ when $w = \overline{w}$

- $w = \overline{w}$ ensures workers are productive ($A = \overline{A}$), else no firm would hire them; but no downward pressure on w because any lower w would make workers unproductive

Search

Idea:

- Workers search for jobs, firms search for workers to fill vacant jobs

- Employment and wages in equilibrium depend on how costly, or difficult, search is

Workers

Notation:

- $p =$ worker's search effort; determines probability of finding job (becoming employable)
- $w =$ wage
- $b =$ unemployment benefit

$pw + (1 - p)b =$ expected income of worker given effort p

$C(p) =$ cost of p

Assume $C'(p) > 0$ and $C''(p) > 0$: effort is costly; more costly on the margin at higher initial effort levels

Utility:

$$U = \underbrace{pw + (1-p)b}_{\text{expected income}} - \underbrace{C(p)}_{\text{cost}}$$

Agents choose p to maximize U, subject to $p \geq 0$ and $p \leq 1$

First-order condition for interior solution:

$$\frac{\partial U}{\partial p} = \underbrace{w - b}_{\text{marg. benefit of effort}} - \underbrace{C'(p)}_{\text{marg. cost of effort}} = 0$$

Corner solutions:
If $w - b < C'(0)$, then optimal to set $p = 0$
If $w - b > C'(1)$, then optimal to set $p = 1$

Why? Illustrate in diagram with p on horizontal axis, and $w - b$ and $C'(p)$ on vertical axis; note that $C''(p) > 0$

Parametric example:

$$C(p) = \frac{a}{2}p^2,$$

where $a > 0$ is an exogenous parameter; higher a means higher marginal cost of effort

Optimal effort:

$$p = \begin{cases} 1 & \text{if} \quad w > a + b \\ (w - b)/a & \text{if} \quad b < w < a + b \\ 0 & \text{if} \quad w < b \end{cases}$$

Interpretation:

Higher wages (w) induce more search effort (p)

If wages as high as $a+b$, then workers exert enough effort to become employable with certainty ($p = 1$)

If wages as low as b, then workers exert no search effort and becomes non-employable with certainty ($p = 0$)

As before, $\overline{L}$ = total number of workers

Labor supply = total number workers who become employable

$$L^S = p\overline{L} = \begin{cases} \overline{L} & \text{if} \quad w \geq a+b \\ \left(\frac{w-b}{a}\right)\overline{L} & \text{if} \quad b < w < a+b \\ 0 & \text{if} \quad w \leq b \end{cases}$$

p is the fraction of all workers who are employable

Think of p as the *labor force participation rate*

The remaining fraction, $1 - p$, are out of the labor force, or "non-employable" (rather than unemployed)

Firms

Firms hire workers by announcing vacancies

Notation:

- V = number of vacancies announced by each firm
- f = fraction vacancies filled
- L = number of workers hired by each firm
- k = cost to announce a vacancy (whether filled or not)

Note that $L = fV$, or:

$$V = \frac{L}{f}$$

Firm profits:

$$\begin{aligned}\pi &= AL^\gamma - wL - kV \\ &= AL^\gamma - wL - k\left(\frac{L}{f}\right) \\ &= AL^\gamma - \left(w + \frac{k}{f}\right)L\end{aligned}$$

Same as before but w replaced by $w + k/f$

- Workers cost both in wages and hiring

- Hiring cost increasing in k, decreasing in f

Setting L to maximize π gives labor demand across all N firms as

$$L^D = N\left(\frac{\gamma A}{w + \frac{k}{f}}\right)^{\frac{1}{1-\gamma}} = N\left(\frac{\gamma A f}{wf + k}\right)^{\frac{1}{1-\gamma}}$$

Illustrate in L^D, L^S in diagram with w on vertical

Draw so that $L^D = L^S < \overline{L}$ in equilibrium

Show effects on equilibrium w and employment (p) from:

(i) increase in a

(ii) increase in k

(iii) increase in f

How about the unemployment rate, u? Technically in this formulation: $u = 0$

Why? Equilibrium determined by $L^S = p\overline{L} = L^D$, so all who are employable find a job in equilibrium by assumption

$$u = \frac{\overbrace{L^S - L^D}^{=0}}{L^S} = 0$$

Here: static model

In dynamic models of search:

- Workers, firms can be both separated and "matched"
- Separation and matching happens randomly over time
- Matches take time to happen; some workers searching for a job at any point in time
- Called *frictional* unemployment

C Intertemporal models

Common tool in modern macro: intertemporal models

Needed to analyze trade-offs over time

E.g., consumption today vs. tomorrow

Examples of fields where intertemporal models are used:

- Business cycle theory, consumption theory: to analyze how much (current) consumption increases in response to increase in income

- Growth theory: what determines the accumulation of (physical and human) capital over time?

- Finance: what determines the return to savings? (Real interest rate, return to risky asset)

More advanced courses: infinite-horizon models

Here: simple two-period model

Agenda:

- Budget constraints
- Preferences
- Utility maximization
- Applications

Budget constraints

Notation:

- C_1 = consumption in period 1
- C_2 = consumption in period 2
- S = saving from period 1 to period 2
- r = (real) interest rate
- Y_1 = income in period 1
- Y_2 = income in period 2

Budget constraint for period 2

$$C_2 = Y_2 + (1 + r)S$$

Budget constraint for period 1

$$C_1 = Y_1 - S$$

Write on intertemporal form: C_2 on left-hand side, C_1 on right-hand side

From budget constraint for period 1: $S = Y_1 - C_1$

Substitute into budget constraint for period 2

$$\begin{aligned} C_2 &= Y_2 + (1 + r)\left[Y_1 - C_1\right] \\ &= \underbrace{Y_2 + (1 + r)Y_1}_{\text{vertical intercept}}\underbrace{-(1 + r)}_{\text{slope}}C_1 \end{aligned}$$

Draw in diagram with C_2 on vertical axis, C_1 on horizontal axis (indifference curve diagram)

Indicate coordinate (Y_1, Y_2)

Where is $S > 0$, $S < 0$, and $S = 0$?

Preferences

Many identical agents

Same as one single *representative agent*

Utility of representative agent depends on C_1 and C_2

Most applied contexts: time-additive utility:

$$U = (1 - \beta)u(C_1) + \beta u(C_2)$$

β measures relative weight on consumption in period 2; $0 < \beta < 1$

(Note: different formulation in problem sets; weights 1 and β)

Functional form for $u(C)$: here logarithmic utility

$$u(C) = \ln(C)$$

Thus:

$$U = (1 - \beta)\ln(C_1) + \beta\ln(C_2)$$

Useful way to understand utility functions: draw indifference curves

Think of C_2 as function of C_1 along the indifference curve

Find slope, $\frac{dC_2}{dC_1}$

Slope of indifference curve (in absolute terms) = Marginal Rate of Substitution (MRS)

Implicitly differentiate

$$(1 - \beta)\frac{1}{C_1} + \beta\frac{1}{C_2}\frac{dC_2}{dC_1} = 0$$

$$\text{slope} = \frac{dC_2}{dC_1} = -\frac{1 - \beta}{\beta}\frac{C_2}{C_1}$$

(Note: different with other utility functions.)

MRS along 45-degree line $= (1 - \beta)/\beta$

Solving the utility maximization problem

Task: maximize $U = (1 - \beta)\ln(C_1) + \beta \ln(C_2)$ subject to budgets constraints

Simplest way:

- Substitute expressions for C_1 and C_2 into utility function; get U as function of S

- Take first-order condition with respect to S

- Solve for S; gives C_1 and C_2

U as function of S:

$$U = (1 - \beta)\ln(Y_1 - S) + \beta \ln(Y_2 + [1 + r]S)$$

First-order condition:

$$\frac{\partial U}{\partial S} = -(1-\beta)\left(\frac{1}{Y_1 - S}\right) + \beta\left(\frac{1+r}{Y_2 + [1+r]S}\right) = 0$$

Solving for S:

$$\begin{aligned}
(1-\beta)\left(\frac{1}{Y_1 - S}\right) &= \beta\left(\frac{1+r}{Y_2 + [1+r]S}\right) \\
(1-\beta)(Y_2 + [1+r]S) &= \beta(Y_1 - S)(1+r) \\
(1-\beta)Y_2 + (1-\beta)(1+r)S &= \beta Y_1(1+r) - \beta S(1+r) \\
(1-\beta)Y_2 + (1+r)S &= \beta Y_1(1+r) \\
(1+r)S &= \beta Y_1(1+r) - (1-\beta)Y_2 \\
S &= \beta Y_1 - (1-\beta)\frac{Y_2}{1+r}
\end{aligned}$$

Finding C_1:

$$\begin{aligned}
C_1 &= Y_1 - S \\
&= Y_1 - \beta Y_1 + (1-\beta)\frac{Y_2}{1+r} \\
&= (1-\beta)\left[Y_1 + \frac{Y_2}{1+r}\right]
\end{aligned}$$

Finding C_2:

$$\begin{aligned}
C_2 &= Y_2 + (1+r)S \\
&= Y_2 + (1+r)\left[\beta Y_1 - (1-\beta)\frac{Y_2}{1+r}\right] \\
&= Y_2 + (1+r)\beta Y_1 - (1-\beta)Y_2 \\
&= \beta\left[(1+r)Y_1 + Y_2\right]
\end{aligned}$$

Define

$$W = Y_1 + \frac{Y_2}{1+r}$$

We call $Y_2/(1+r)$ the present value of period-2 income

Now we can write optimal C_1 and C_2 as

$$\begin{aligned}
C_1 &= (1-\beta)W \\
C_2 &= \beta(1+r)W
\end{aligned}$$

Show graphs of C_1 and C_2 in diagram with W on horizontal axis; called *Engel curves*

Illustrate W and $(1+r)W$ in the indifference curve diagram (with C_1 and C_2 on the axes)

Indifference curve tangent to budget line at $C_1 = (1-\beta)W$, $C_2 = \beta(1+r)W$

Insight: same results as if income in period 1 is W, and income in period 2 is zero.

Some problems:

- Illustrate how the optimal choices of C_1 and C_2 change as β increases. Answer: move up the budget line.

- Illustrate how the optimal choices of C_1 and C_2 change as W increases. Answer: budget line shifts out; C_1 and C_2 increase but ratio $C_2/C_1 = (1+r)\beta/(1-\beta)$ unchanged.

- How do the optimal choices of C_1 and C_2 change if Y_1 and Y_2 change, but W is unchanged? Answer: not at all (called *Ricardian Equivalence*.)

- Find optimal S, C_1, and C_2 when

$$u(C) = \frac{C^{1-\theta}}{1-\theta}$$

where $\theta > 0$

Application I: a closed economy without production

Now slightly different notation:

- $Y_1 = Y =$ income in period 1

- $Y_2 = (1 + g)Y =$ income in period 2

- $g =$ growth rate of income, i.e.,

$$g = \frac{Y_2 - Y_1}{Y_1}$$

Here: g and Y exogenous parameters; r in endogenous

Closed economy: no borrowing or lending overseas

Task: find the equilibrium interest rate (the only endogenous variable of interest)

All agents identical, face same r, so they must choose the same S

Equilibrium condition: representative agent sets $S = 0$

Why?

- Closed economy, so anyone who borrows must borrow from someone else in the same economy

- But if one agent borrows, so does everyone else (recall: same S)

- Thus, net borrowing must be zero

Set $S = 0$:

$$\begin{aligned} S &= \beta Y_1 - (1-\beta)\frac{Y_2}{1+r} \\ &= \beta Y - (1-\beta)\frac{Y(1+g)}{1+r} = 0 \end{aligned}$$

Solve for r:

$$r = \frac{(1-\beta)(1+g)}{\beta} - 1$$

Insights:

- Faster growth (higher g) leads to higher r. Intuition: when future incomes high, agents want to borrow; r adjusts so that $S = 0$

- r does not depend on period-1 income, Y. Only income in period 1 relative to period 2 matters. (Due to log utility)

This model describes an *exchange economy*: g exogenous; Y_1 and Y_2 are fixed endowments to be traded

Next: allow for *production* and *investment*, making g and r endogenous

Application II: a closed economy with production

Notation:

$K =$ capital used in production in the second period

$AK^\alpha =$ total amount produced in the second period (GDP)
$\alpha \in (0,1)$, $A > 0$ are parameters from production function; see below

$Y_1 =$ labor income in the first period (here exogenous)

$Y_2 =$ labor income in the second period $= (1-\alpha)AK^\alpha$
$1-\alpha$ is known as the labor share of output; α the capital share of output

Firms

Firms choose K to max profits:

$$\pi = AK^\alpha - (1+r)K$$

First-order condition:

$$\frac{\partial \pi}{\partial K} = A\alpha K^{\alpha-1} - (1+r) = 0$$

Use $Y_2 = (1-\alpha)AK^{\alpha}$ and $1+r = A\alpha K^{\alpha-1}$ to write

$$\frac{Y_2}{1+r} = \left(\frac{1-\alpha}{\alpha}\right) K$$

Illustrate in diagram with $Y_2/(1+r)$ on vertical axis, K on horizontal axis

Can be interpreted as demand for capital

Agents

Agents maximize same old log utility: $U = (1-\beta)\ln(C_1) + \beta \ln(C_2)$

Gives optimal saving:

$$S = \beta Y_1 - (1-\beta)\frac{Y_2}{1+r}$$

Or:

$$\frac{Y_2}{1+r} = \frac{\beta Y_1 - S}{1-\beta}$$

Illustrate in diagram with $Y_2/(1+r)$ on vertical axis, S on horizontal axis

Can be interpreted as supply of capital

Capital market equilibrium

Capital market equilibrium requires demand for capital equals supply: $K = S$

Illustrate in three stacked diagrams:

- Top: $Y_2/(1+r)$ on vertical axis, S and K on horizontal axis

- Middle: AK^{α} and $Y_2 = (1-\alpha)AK^{\alpha}$ on vertical axis, K on horizontal axis

- Bottom: $1+r = A\alpha K^{\alpha-1}$ on vertical axis, K on horizontal axis

Analyze effects of:

- Increase in β (higher K, AK^{α}, and Y_2; lower r)

- Increase in A (no change in K; higher AK^{α}, Y_2 and r)

Application III: an open economy with production

Same notation as for closed economy

News: r^* = exogenous world interest rate

Firms

Can borrow as much as they want at rate r^*

Firms choose K to max profits:

$$\pi = AK^\alpha - (1 + r^*)K$$

First-order condition:

$$\frac{\partial \pi}{\partial K} = A\alpha K^{\alpha-1} - (1 + r^*) = 0$$

Let K^* denote firms' optimal choice of K given r^*

$$K^* = \left(\frac{\alpha A}{1 + r^*}\right)^{\frac{1}{1-\alpha}}$$

Note: agents' savings (S) or preferences (e.g. β) play not role

Intuition: firms have access to international capital markets; domestic saving not needed for investment

Let Y_2^* denote the level of $Y_2 = (1-\alpha)AK^\alpha$ associated with $K = K^*$

$$Y_2^* = (1-\alpha)A(K^*)^\alpha$$

Set $K = K^*$ in FOC: $1 + r^* = \alpha A(K^*)^{\alpha-1}$

Gives present-value of second period income

$$\frac{Y_2^*}{1+r^*} = \frac{(1-\alpha)A(K^*)^\alpha}{\alpha A(K^*)^{\alpha-1}} = \left(\frac{1-\alpha}{\alpha}\right)K^*$$

Use expression for K^* above

$$\frac{Y_2^*}{1+r^*} = \left(\frac{1-\alpha}{\alpha}\right)K^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha A}{1+r^*}\right)^{\frac{1}{1-\alpha}}$$

Gives $Y_2^*/(1+r^*)$ as function of exogenous variables: r^* and parameters of production function (α and A)

Draw in diagram with r^* in horizontal axis

Agents

Agents maximize same old log utility: $U = (1-\beta)\ln(C_1) + \beta \ln(C_2)$

Here:

- Interest rate $= r^*$
- Present value of second period income $= Y_2^*/(1+r^*)$

Optimal saving:

$$S = \beta Y_1 - (1-\beta)\frac{Y_2^*}{1+r^*}$$

Or:

$$\frac{Y_2^*}{1+r^*} = \frac{\beta Y_1 - S}{1-\beta}$$

The trade balance

National Accounts identity for the first period

$$Y_1 = C_1 + K^* + NX_1$$

In words: GDP=Consumption+Investment+Gov't spending+Trade balance

Here:

- No government spending, $G = 0$
- $K^* =$ investment
- $NX_1 =$ net export (trade balance) in first period

Recall that $S = Y_1 - C_1$

$$NX_1 = S - K^*$$

Use two diagrams stacked beside each other to determine NX_1

- First diagram (to the left): r^* on horizontal axis; $Y_2^*/(1+r^*)$ on vertical axis. Illustrate

$$\frac{Y_2^*}{1+r^*} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha A}{1+r^*}\right)^{\frac{1}{1-\alpha}}$$

- Second diagram (to the right): S and K^* on horizontal axis; $Y_2^*/(1+r^*)$ on vertical axis. Illustrate

$$\frac{Y_2^*}{1+r^*} = \frac{\beta Y_1 - S}{1-\beta}$$

$$\frac{Y_2^*}{1+r^*} = \left(\frac{1-\alpha}{\alpha}\right) K^*$$

Pick some r^* in left-hand diagram

This gives $Y_2^*/(1 + r^*)$

Read off $NX_1 = S - K^*$ in right-hand diagram

Analyze effects on S, K^*, NX_1 from

- Increase in r^*

- Increase in A

- Increase in β

How about trade balance in second period?

Exercise (see problems): show that

$$NX_2 = -(1 + r^*)NX_1$$

Trade deficit today paid back with interest by running surplus tomorrow; vice versa

Application IV: open economy without production, but with credit market imperfections

Same notation as in Application I

$Y_1 = Y =$ income in period 1

$Y_2 = (1+g)Y =$ income in period 2; $g =$ growth rate of income

World interest rate, r^*, exogenous (small open economy)

Utility same as before

News: some agents (but not all) can *default* on their debt

To default on debt means not paying it back to the lender

Here: an exogenous fraction $1-a$ of the agents default; the reminder (a) pay back all their debts; $a \in (0,1)$

(Other models: choice to default endogenous)

Banks

A bank is a *financial intermediary*, which means it borrows from one source, lends to another

Here: banks borrow on the world market at exogenous rate r^*, lend out to agents in the home economy at (endogenous) rate $\widehat{r}$

Bank's profit (per unit borrowed/lent):

$$\pi = a \times (1 + \widehat{r}) + (1 - a) \times 0 - (1 + r^*)$$

Revenue from those that pay back: $a(1 + \widehat{r})$

Revenue from those that default: 0

Cost of borrowing: $1 + r^*$

Free entry into banking sector means all banks make zero profit: $\pi = 0$

Thus:

$$1 + \widehat{r} = \frac{1 + r^*}{a} > 1 + r^*$$

Difference between $\widehat{r}$ and r^* called *default premium*

Agents

Agents face different interest rates depending on whether they are borrowers ($S < 0$) or lenders ($S > 0$)

Lenders face the rate r^*, borrowers the rate $\widehat{r} > r^*$

Illustrate in indifference curve diagram

C_1 and C_2 on axes

Budget line kinked at $C_1 = Y$, $C_2 = Y(1 + g)$

Slope: $-(1 + r^*)$ above kink; $-(1 + \widehat{r})$ below kink

Here all agents are the same; all borrow or all lend

As before, saving equals:

$$S = \left[\beta - (1-\beta)\frac{(1+g)}{1+r}\right] Y$$

where now

$$1+r = \begin{cases} 1+r^* & \text{if } S \geq 0, \\ 1+\widehat{r} = \frac{1+r^*}{a} & \text{if } S < 0. \end{cases}$$

Saving and interest rate jointly determined; depend on one another

Illustrate how outcome depends on exogenous parameters (a, β, g, and r^*)

Draw line where $1+r^*$ takes different (positive) values

Indicate the points $\frac{a(1-\beta)(1+g)}{\beta}$ and $\frac{(1-\beta)(1+g)}{\beta}$ on the line

Show where $S > 0$, $S < 0$, and $S = 0$

Case 1: $1 + r^* > \frac{(1-\beta)(1+g)}{\beta}$

Means $S > 0$ even when $1 + r = 1 + r^*$

World interest rate so high everyone prefers to save

Case 2: $1 + r^* < \frac{a(1-\beta)(1+g)}{\beta}$

Means $S < 0$ even when $1 + r = \frac{1+r^*}{a} = 1 + \widehat{r}$

Everyone prefers to borrow even at rate $\widehat{r}$

Case 3: $\frac{a(1-\beta)(1+g)}{\beta} < 1 + r^* < \frac{(1-\beta)(1+g)}{\beta}$

Means $S = 0$. Agents do not borrow, nor lend

Exercise: For Case 3 above, show agent's optimal choice in an indifference curve diagram

Discussion:

Banks would prefer not to lend to those who will not pay back (the defaulters), but it has no way of knowing who they are. They only know their fraction of the population (a)

Agents themselves may, or may not, know whether they are defaulters

Those who are defaulters have no incentive to tell, and those who are not have no way of proving they will pay back

One economic actor knowing more than another called *asymmetric information*

If all actors know equally much/little: *imperfect information*

In many applications, same results with asymmetric as with imperfect information

In richer models:

- Agents may choose whether or not to default, and/or what actions to take to avoid default
- Banks may undertake actions to learn who will default or not, e.g., by keeping records of credit histories; gives incentive to agents not to default
- Banks may lend only to agents who (through a written contract) let the bank seize property belonging a defaulting borrower; e.g. a house or a car. Such property is called *collateral*

Sovereign default risks

In the above application the borrower was an agent and the lender a bank.

In similar models the borrower could be a government and the lenders could be actors on the international credit markets (e.g., pension funds)

When governments of countries default it is sometimes called sovereign default

When there is risk of sovereign default governments often pay a default premium, which can affect the interest rates faced by domestic firms

- Interesting implications for fiscal policy

- Cutting government spending and/or rasing taxes can lower the risk of default, thus lower default premium, and the domastic interest rate, which in turn can increase investment

D Monetary policy, rational expectations, and dynamic consistency

Often debated issue in macroeconomics: what are the real effects of monetary (or fiscal) policy?

Is there a case for active monetary (or fiscal) stabilization policy?

Goal here: understanding some concepts, e.g., Rational Expectations, Lucas Critique, Dynamic (or Time) Consistency, Money Neutrality, Commitment and Discretion

Notation:

- P denotes general price level
- M denotes nominal money supply ($)
- Y denotes output, or income (GDP) $=$ number of transactions
- V denotes the *velocity* of money $=$ number of times a $ bill is used in transactions

When a change in M affects Y it is called that *monetary policy has real effects*

Effects on P are not real effects, but *nominal effects*

When monetary policy has no real effects we say that *money (*or *monetary policy) is neutral*

Assume that all goods that are produced are bought and sold once

Follows logically that both PY and MV denote the \$ value of all transactions

Gives the *Quantity Equation*:

$$MV = PY$$

Let smaller case letters denote logs:

$$\ln M + \ln V = \ln P + \ln Y$$
$$m + v = p + y$$

Rename v to y^* and rearrange:

$$y = y^* + m - p$$

Called *Aggregate Demand (AD) function*

(Can also be derived from IS-LM model)

An increase in (log) real money supply, $m - p$, raises output, above its long-run equilibrium level, y^*

Sometimes add a parameter that measures the effect on output from real money shocks; here called ϕ

$$y = y^* + \phi\,(m - p)$$

Draw in diagram with y on horizontal axis, p on vertical

Insight from this AD function: a change in m at given p leads to change in y

Crucial question: what determines p?

Three theories:

I. Fixed prices

II. Flexible prices

III. Predetermined prices

From here on: m is a *stochastic (random) variable*

Theories about how prices are determined

I. Fixed prices

p exogenous, here meaning a non-stochastic constant

Since $y = y^* + \phi\,(m - p)$ it follows trivially that:

$$m \uparrow \implies \begin{cases} y \uparrow \\ p \text{ unchanged} \end{cases}$$

Monetary policy has real effects (effect on y)

Note also:

$$\begin{aligned} E(y) &= y^* + \phi\,[E(m) - p] \\ V(y) &= \phi^2 V(m) \end{aligned}$$

Changes in $E(m)$ lead to changes in $E(y)$

Changes in $V(m)$ lead to changes in $V(y)$

II. Flexible prices

p endogenous, set by firms

Firms being flexible means they can adjust p to changes in m

Timing of events:

1. m is realized

2. firms set prices

3. y is determined

Many firms; let p_i denote the price set by some small firm i

Takes the general price level, p, as given

Assumption about firms' price-setting behavior

$$p_i = ap + (1 - a)m$$

where $0 < a < 1$

This price-setting behavior can be derived from richer models, where firms set prices under monopolistic competition

Here not setting up such a model; enough to understand intuition:

- higher p means costs of inputs higher for each firm: then the level of p_i that maximizes profits is higher

- higher m means higher demand from consumers: firms can charge higher prices; profit-maximizing p_i higher

In equilibirum all firms are identical; set same prices

General price level, p, determined by equilibrium condition:

$$p_i = p$$

This gives:

$$\begin{aligned} p &= ap + (1-a)m \\ p &= m \end{aligned}$$

Price level changes one-to-one with money supply

Recall: $y = y^* + \phi\,(m - p)$

Thus, $y = y^*$, regardless of m

$$m \uparrow \;\Longrightarrow\; \begin{cases} y \text{ unchanged} \\ p \uparrow \end{cases}$$

Monetary policy has no *real* effects (no effects on y), only *nominal* effects (effects on p); money is *neutral*

Note also:

$$\begin{aligned} E(y) &= y = y^* \\ V(y) &= V(y^*) = 0 \end{aligned}$$

Changes in $E(m)$ have no effect on $E(y)$

Changes in $V(m)$ have no effect on $V(y)$

III. Predetermined prices

p endogenous, set by firms, but before m is realized

Timing:

1. firms set prices

2. m is realized

3. y is determined

Firms' pricing similar as under II, but now p_i set only knowing $E(m)$, not m

$$p_i = ap + (1-a)E(m)$$

Pause for some reflections:

- Here we assume that firms set prices as if m is equal to $E(m)$ with certainty: called *Certainty-Equivalence* behavior
  - Different results if we assume that they maximize *expected* profits (or expected utility); then prices could depend also on e.g. $V(m)$

- Expectations formed under the *objective probability distribution;* firms have *Rational Expectations*
  - Alternative assumptions: firms may use the lowest (or highest) value m can take, or assign other subjective probabilities (see problem set)
  - Non-rational expectations often problematic when setting up models: no discipline imposed, assumptions arbitrary

Same equilibrium condition:

$$p_i = p$$

This gives:

$$p = E(m)$$

Note: m need not equal $E(m)$, so p can be different from m

Recall: $y = y^* + \phi\,(m - p)$

Thus, $y = y^* + \phi\,[m - E(m)]$

$$m \uparrow \text{ while } E(m) \text{ unchanged} \Longrightarrow \begin{cases} y \uparrow \\ p \text{ unchanged} \end{cases}$$

$$m \uparrow \text{ and } E(m) \uparrow \text{ by same amount} \Longrightarrow \begin{cases} y \text{ unchanged} \\ p \uparrow \end{cases}$$

Conclusions from model with predetermined prices:

- *Anticipated* (expected) monetary policy is neutral
- *Unanticipated* (unexpected) monetary policy has real effects

Note also:

$$\begin{aligned} E(y) &= E(y^* + \phi\,[m - E(m)]) = y^* \\ V(y) &= V(y^* + \phi\,[m - E(m)]) = \phi^2 V(m) \end{aligned}$$

Changes in $E(m)$ have no effect on $E(y)$

Changes in $V(m)$ lead to changes in $V(y)$

Discussion:

- If we observe that unexpected changes in m have real effects, should policy markers pursue active monetary policy?

- Not necessarily, because the relationship is observed at given $E(m)$

- If agents are rational, a change in policy would lead to change in $E(m)$

- Called the *Lucas Critique*

Dynamic (or Time) Consistency

Consider a model of the labor market, with nominal wages and prices

- Workers want real wages to equal some target level
- Set/negotiate nominal wages taking into account what they expect about price level, which is set by Central Bank (CB) through monetary policy (by printing money)

More formally: workers set w^{NOM} such that expected real wages equal target level, denoted $\widehat{w}$; here treated as exogenous:

$$w^{NOM} = \widehat{w} P^e$$

where P^e is the expected price level

Note:

- We do not write $E(P)$, because P is not stochastic

- In equilibrium, the actual price level (P) always equals the expected price level (P^e)
  - This is the same as *Rational Expectations*

Firms maximize profits, taking nominal wages as given. This gives labor demand by each firm as

$$L = \left(\frac{\gamma AP}{w^{NOM}}\right)^{\frac{1}{1-\gamma}}$$

(see problem sets)

Let $Y = NAL^{\gamma}$ be total output by all N firms (same as GDP)

Using expressions above we get

$$Y = NA\left(\frac{\gamma AP}{\widehat{w}P^e}\right)^{\frac{\gamma}{1-\gamma}} = NA^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{\widehat{w}}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{P}{P^e}\right)^{\frac{\gamma}{1-\gamma}}$$

Define

$$
\begin{aligned}
Y^* &= NA^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\widehat{w}}\right)^{\frac{\gamma}{1-\gamma}} \\
\phi &= \frac{\gamma}{1-\gamma}
\end{aligned}
$$

Let P_0 be some initial (known) price level, and let π and π^e be the actual and expected rates of inflation (=percentage change in prices)

$$
\begin{aligned}
P &= P_0(1+\pi) \\
P^e &= P_0(1+\pi^e)
\end{aligned}
$$

Now it follows that

$$
Y = Y^* \left(\frac{P}{P^e}\right)^{\phi} = Y^* \left(\frac{1+\pi}{1+\pi^e}\right)^{\phi}
$$

$$
\ln(Y) = \ln(Y^*) + \phi \left[\overbrace{\ln(1+\pi)}^{\simeq \pi} - \overbrace{\ln(1+\pi^e)}^{\simeq \pi^e} \right]
$$

Letting lower-case variables be logs gives the so-called the *Phillips Curve:*

$$y = y^* + \phi(\pi - \pi^e)$$

Illustrate Phillips Curve in diagram with y on horizontal axis, π on vertical

In equilibrium, when inflation is what workers expect it to be, (log) GDP equals y^*; referred to as the equilibirum output

Taking π^e as given, the CB may want to increase π (by printing money) in an attempt to raise y above y^*

May not succeed but the very temptation can matter for inflation outcomes

This is the essence of *Dynamic Inconsistency*

Next: capture this in a formal model

- CB acts as a rational agent maximizing an objective function
- CB desires higher output than the equilibrium level
- Phillips Curve works like a budget constraint for the CB
- Outcome different depending on timing of events, i.e., whether or not the CB can *commit* himself to a policy

CB's preferences

Minimizes a "loss function," L

$$L = \frac{1}{2}(y - \widetilde{y})^2 + \frac{\alpha}{2}(\pi - \widetilde{\pi})^2$$

Idea: higher L, bigger loss

Same result if defining utility as $U = -L$; here minimizing loss, instead of maximizing utility

Notation:

- $\widetilde{y} =$ CB's most desired level of output
- $\widetilde{\pi} =$ CB's most desired level of inflation
- $\alpha =$ CB's weight on inflation; higher α means CB "hates" inflation more

Assumption: $\widetilde{y} > y^*$

CB prefers higher output than equilibrium

Draw indifference curves in diagram with y on horizontal axis, π on vertical

Two sets of assumption about timing

Commitment:

1. CB sets π
2. Public forms expectations (sets π^e)
3. Output is determined

Discretion:

1. Public forms expectations (sets π^e)
2. CB sets π
3. Output is determined

Commitment

CB sets π before public forms expectation

Thus the public knows π

Thus $\pi^e = \pi$

Thus $y = y^* + \phi(\pi - \pi^e) = y^*$, regardless how CB sets π

So what is the CB's optimal choice of π?

Simply insert $y = y^*$ into L

$$L = \frac{1}{2}(y^* - \widetilde{y})^2 + \frac{\alpha}{2}(\pi - \widetilde{\pi})^2$$

First-order condition for π:

$$\alpha\left(\pi-\widetilde{\pi}\right)=0 \implies \pi=\widetilde{\pi}$$

Outcome under commitment (denote by super-index C):

$$\pi^{C}=\widetilde{\pi}$$

$$y^{C}=y^{*}$$

Discretion

CB sets π after public forms expectation, taking as given the public's expectations (π^e)

Output given by $y = y^* + \phi(\pi - \pi^e)$

So what is the CB's optimal choice of π?

Insert $y = y^* + \phi(\pi - \pi^e)$ into L

$$L = \frac{1}{2}\left(y^* + \phi\left[\pi - \pi^e\right] - \widetilde{y}\right)^2 + \frac{\alpha}{2}\left(\pi - \widetilde{\pi}\right)^2$$

First-order condition for π:

$$\frac{\partial L}{\partial \pi} = \left(y^* + \phi\left[\pi - \pi^e\right] - \widetilde{y}\right)\phi + \alpha\left(\pi - \widetilde{\pi}\right) = 0$$

Solve for π:

$$\pi = \left[\frac{\phi(\widetilde{y} - y^*) + \alpha\widetilde{\pi}}{\phi^2 + \alpha}\right] + \left(\frac{\phi^2}{\phi^2 + \alpha}\right)\pi^e$$

Gives the CB's choice of π as function of public's expectations, π^e, and exogenous parameters

Draw in diagram with π on vertical axis, π^e on horizontal axis

No uncertainty, no stochastic variables: if public has rational expectations π^e must equal π given above

Equilibrium condition:

$$\pi^e = \pi = \left[\frac{\phi(\widetilde{y} - y^*) + \alpha\widetilde{\pi}}{\phi^2 + \alpha}\right] + \left(\frac{\phi^2}{\phi^2 + \alpha}\right)\pi$$

Note that we impose equilibrium condition ($\pi^e = \pi$) *after* having solved for CB's optimal π

Solve for π as function of exogenous parameters only:

$$\pi = \frac{\phi\overbrace{(\widetilde{y} - y^*)}^{>0}}{\alpha} + \widetilde{\pi}$$

Output given by: $y = y^* + \phi(\pi - \pi^e) = y^*$

Outcome under discretion (denote by super-index EQ):

$$\pi^{EQ} = \frac{\phi(\widetilde{y} - y^*)}{\alpha} + \widetilde{\pi}$$

$$y^{EQ} = y^*$$

Discussion

Output same under discretion as under commitment ($y^{EQ} = y^C = y^*$)

Inflation higher under discretion than commitment ($\pi^{EQ} > \pi^C$)

Why?

Under discretion CB has credibility problem

CB could announce low π before π^e is set (*ex ante*)

Once expectations are set (*ex post*) CB tempted to increase y by raising π a little

Public knows CB faces this temptation and adjusts expectations accordingly

π must be high enough so as to not tempt CB to raise π further

Illustrate in indifference curve diagram: π on vertical axis, y on horizontal

Is CB better off setting policy under commitment or discretion?

Answer obvious:

- Output the same, but worse inflation outcome under discretion

- CB better off under commitment than discretion

One may think more power to change policy (as under discretion) would make CB better off

Not here: being "tied to the mast" (as under commitment) makes the CB better off

Benefits of commitment comes from overcoming credibility problem

Other applications of dynamic consistency problems:

- Capital taxation:
  - Government announces low taxation
  - Agents/firms undertake investments
  - Ex post, government has an incentive to deviate, raise taxes
  - Anticipating this, agents/firms may choose not to investment

- Fixed exchange rates:
  - Government (or CB) announces fixed exchange rate – not to let $ depreciate
  - Workers in export sector (or their unions) set wages (or make wage demands)
  - Ex post, government has an incentive to deviate, let $ depreciate, thus boosting exports and employment in export sector, at the expense of workers' real wages
  - Anticipating this, workers/unions may raise wage demands

- Petitions at York (also related to Samaritan's Dilemma)
  - Administrators pity students in trouble
  - Save them in various ways
  - Students expect lenient treatment, exert less effort

E Political Economy

In most macroeconomic models, economics policy is treated as exogenous

Examples: taxes, government spending, money supply usually exogenous

Exception: in the model of dynamic consistency of monetary policy, policy was derived from the decision problem of the policymaker (CB)

Other models: government may care about consumption of different agents; redistributive policies derived endogenously (not here)

Now take this one step further: derive endogenously who gets to be the policymaker – set up a model of *Electoral Competition*

For the moment: forget about policy altogether; only interested in what candidate wins and how; not what (s)he does after winning

A Model of Electoral Competition

The electorate (set of voters)

- Each voter has some political preference denoted x
- Represented by a point on the (continuous) interval $[0, 1]$ (only one political dimension)
- x close to 0 means relatively "left," x close to 1 relatively "right"
- Mass of voters at x denoted $f(x)$
- $f(x)$ called *probability density function* (pdf)
- Here uniform distribution: $f(x) = 1$ for all $x \in [0, 1]$; 0 elsewhere

Illustrate in diagram with x on horizontal axis, $f(x)$ on vertical

Area under the whole pdf equals one

$$\int_0^1 f(x)dx = 1$$

Area under pdf to the left of some point $\widehat{x}$ equals $\widehat{x}$

$$\int_0^{\widehat{x}} f(x)dx = \widehat{x}$$

Convenient feature of the model:

- $\widehat{x}$ = fraction of the electorate positioned to the left of $\widehat{x}$

- $1 - \widehat{x}$ = fraction of the electorate positioned to the right of $\widehat{x}$

- Can illustrate everything on the $[0, 1]$ line

- What x is most "centrist" (equal fractions to the left as to the right)? (Answer: $x = 1/2$. Called median voter.)

The candidates

- Each voter has one vote
- Vote for either one of two candidates, denoted by 1 and 2
- Candidates hold positions on $[0, 1]$
- Positions denoted a_1 and a_2
- Vote shares denoted S_1 and S_2
- Majority rule: candidate with most votes wins; equal probability (1/2) if equal vote shares ($S_1 = S_2$)

Reminder of the presentation:

1. Given the positions of the two candidates, how do voters vote?

2. Which candidate wins the election?

3. How do the two candidates position themselves? (Answer: at the midpoint, $a_1 = a_2 = 1/2$. Called the *Median Voter Theorem*.)

How do voters vote?

Assumptions:

- Each voter votes for the candidate who is closest

- If both candidates hold the same position $(a_1 = a_2)$, voters flip a coin

Pick any two points for a_1 and a_2 on $[0, 1]$

What are their vote shares?

Recall: voters go to the candidate that is closest

Outcome easy to see on the $[0, 1]$ line

- If $a_1 < a_2$:
  - Everyone to the left of 1 vote for 1
  - Everyone to the right of 2 vote for 2
  - Candidates split vote in between

$$S_1 = a_1 + \frac{a_2 - a_1}{2} = \frac{a_1 + a_2}{2}$$

$$S_2 = 1 - a_2 + \frac{a_2 - a_1}{2} = 1 - \frac{a_1 + a_2}{2}$$

- If $a_1 > a_2$:
  - Everyone to the left of 2 vote for 2
  - Everyone to the right of 1 vote for 1
  - Candidates split vote in between

$$S_1 = 1 - a_1 + \frac{a_1 - a_2}{2} = 1 - \frac{a_1 + a_2}{2}$$

$$S_2 = a_2 + \frac{a_1 - a_2}{2} = \frac{a_1 + a_2}{2}$$

- If $a_1 = a_2$:
  - Candidates split electoral vote equally between them

$$S_1 = S_2 = \frac{1}{2}$$

Which candidate wins the election?

Assumptions:

- If $S_1 > S_2$, then 1 wins
- If $S_2 > S_1$, then 2 wins
- If $S_2 = S_1$, then both win with probability $1/2$

More formally, let p denote the probability that 1 wins (and $1 - p$ the probability that 2 wins)

$$p = \begin{cases} 1 & \text{if } S_1 > S_2 \\ \frac{1}{2} & \text{if } S_1 = S_2 \\ 0 & \text{if } S_1 < S_2 \end{cases}$$

Recall:

$$S_1 = \begin{cases} \frac{a_1+a_2}{2} & \text{if } a_1 < a_2 \\ \frac{1}{2} & \text{if } a_1 = a_2 \\ 1 - \frac{a_1+a_2}{2} & \text{if } a_1 > a_2 \end{cases}$$

$$S_2 = \begin{cases} 1 - \frac{a_1+a_2}{2} & \text{if } a_1 < a_2 \\ \frac{1}{2} & \text{if } a_1 = a_2 \\ \frac{a_1+a_2}{2} & \text{if } a_1 > a_2 \end{cases}$$

Task: find condition in terms of a_1 and a_2, under which $S_1 > S_2$, $S_1 = S_2$, and $S_1 < S_2$

Note:

$$\frac{a_1 + a_2}{2} > (<, =)\frac{1}{2} \iff a_2 + a_1 > (<, =)1$$

Two inequalities to keep track of:

- Which is greater, a_1 or a_2?

- Is $a_1 + a_2$ greater or smaller than one?

Illustrate in diagram with a_1 on horizontal axis, a_2 on vertical; note both lie on interval $[0, 1]$

Show where a_1 is greater/smaller than a_2; different sides of the line $a_2 = a_1$

Show where $a_1 + a_2$ is greater/smaller than 1; different sides of the line $a_2 = 1 - a_1$

These lines divide the diagram into four fields

Illustrate five cases in the diagram:

Case A: $a_1 < a_2$, $a_1 + a_2 > 1$
$S_1 = (a_1 + a_2)/2 > 1/2$
$p = 1$, 1 wins

Case B: $a_1 > a_2$, $a_1 + a_2 > 1$.
$S_2 = (a_1 + a_2)/2 > 1/2$
$p = 0$, 2 wins!

Case C: $a_1 > a_2$, $a_1 + a_2 < 1$
$S_1 = 1 - (a_1 + a_2)/2 > 1/2$
$p = 1$, 1 wins

Case D: $a_1 < a_2$, $a_1 + a_2 < 1$
$S_2 = 1 - (a_1 + a_2)/2 > 1/2$
$p = 0$, 2 wins

Case E: $a_1 = a_2 = 1/2$, $a_1 + a_2 = 1$
$S_1 = (a_1 + a_2)/2 = 1/2$
$p = 1/2$, 1 and 2 win with equal probability!

How do the two candidates position themselves?

Assumption: candidates purely office motivated, care only about winning

(Alternative assumption: care also about ideology; see problem set)

Equilibrium concept used here: *Nash equilibrium*

Means (roughly) that no *one-sided deviation* can improve any of the candidate's chances of winning

Median Voter Theorem (MVT):

$a_1 = a_2 = 1/2$ is the only Nash equilibrium

"Proof:"

(1) Show that $a_1 = a_2 = 1/2$ is a Nash equilibrium (existence)

- If 1 chooses $a_1 = 1/2$, then choosing $a_2 = 1/2$ is the best 2 can do; vice versa for 1

- Why? If $a_1 = a_2 = 1/2$, then $p = 1 - p = 1/2$; any one-sided deviation means certain defeat

(2) Show it is the *only* Nash equilibrium (uniqueness)

- Pick any of the cases A to D in the diagram with a_1 and a_2 on the axes; either 1 or 2 wins; loser can always change outcome through a one-sided deviation ("leapfrogging")

- If in A or C, candidate 2 can move from certain defeat to certain victory; if in B or D, candidate 1 can move from certain defeat to certain victory

Discussion:

- Model used here very simple
- However, MVT can be derived from richer models (i.e., under less restrictive assumptions)
- For example, holds also with non-uniform distribution of voter preferences
- Useful when deciding how to model preferences of the policymaker: identical to the median voter

Let $f(x)$ be the probability density function describing of voter preferences

Same support as before: $[0, 1]$

Let the cumulative density function be

$$F(\widehat{x}) = \int_0^{\widehat{x}} f(x)dx$$

The fraction of the voters to the left of $\widehat{x}$ equals $F(\widehat{x})$

Median voter's preference, call it m, is given by $F(m) = 1/2$

Illustrate in two different diagrams: both with x on horizontal axis; $f(x)$ or $F(x)$ on the vertical axes