Midterm Exam – Econ 4020
February 10, 2025
Department of Economics
York University

**Instructions:** On this exam, you should *not* show how you arrived at your answer, only write the answer in the space indicated on the relevant answer sheet. Answers must be given on the correct answer sheet, one for each problem. If you make a mistake, ask for a new answer sheet. Do not fold the answer sheets or write on the back, or outside of the indicated space.

Student name: SID number:

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**1. The Solow Model I [4 marks]**

Consider a Solow model with Cobb-Douglas production. The capital stock per effective worker, $k$, evolves over time according to

$$\dot{k} = sk^{1-\lambda} - (n + g + \delta)k,$$

where the parameters $\lambda$, $s$, $n$, $g$, and $\delta$ are strictly positive and constant, and $0 < \lambda < 1$ and $0 < s < 1$. Let $y = k^{1-\lambda}$ be output per effective worker, and let $k^*$ and $y^*$ be the steady-state levels of $k$ and $y$, respectively.

**(a)** Find $k^*$ in terms of (some or all of) $\lambda$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(b)** Find $k^*/y^*$ in terms of (some or all of) $\lambda$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(c)** Let $r = \frac{\partial y}{\partial k} - \delta$. Find an expression for the steady-state level of $r$, denoted $r^*$. Your answer should be in terms of (some or all of) $\lambda$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(d)** In the diagram provided, indicate $\ln(s)$ on the horizontal axis and $\ln(y^*)$ on the vertical axis and draw the graph of $\ln(y^*)$. Indicate slope. [1 mark]

**2. The Solow Model II [4 marks]**

Consider a standard Solow model, where the capital stock per effective worker, $k$, evolves over time according to

$$\dot{k} = sf(k) - (n + g + \delta)k.$$

Here $f(k)$ is a general intensive-form production function, such that $f(0) = 0$, $f'(k) > 0$ and $f''(k) < 0$. As in class, we let $c = (1 - s)f(k)$ be consumption per effective worker, where $s$ is the rate of saving (or investment), such that $0 < s < 1$. The parameters $n$, $g$, and $\delta$ are strictly positive and constant.

Assume that the economy is initially in a steady state associated with $s = s_0$. Then $s$ changes from $s_0$ to $s_1$, from some point in time $\widehat{t}$, and then stays at $s_1$ forever. We also assume that

$$0 < s_0 < s_1 < s_{\text{GR}} < 1,$$

where $s_{\text{GR}}$ is the golden-rule level of $s$, which maximizes steady-state consumption per effective worker, $c^*$.

Let $k_0^*$ and $k_1^*$ be the steady-state levels of $k$ associated with $s_0$ and $s_1$, respectively. Similarly, let $c_0^*$ and $c_1^*$ be the steady-state levels of $c$ associated with $s_0$ and $s_1$, respectively.

**(a)** In the space provided, draw a break-even investment diagram. For full mark you must: (i) indicate $k$ on the horizontal axis; (ii) draw the graphs of $s_0f(k)$, $s_1f(k)$ and $f(k)$ correctly; (iii) draw the graph of $(n + g + \delta)k$ correctly; (iv) indicate $k_0^*$ and $k_1^*$ on relevant axis; and (v) show how to read off $c_0^*$ and $c_1^*$. [2 marks]

**(b)** In the diagram provided, draw the time path for $c$. For full mark you must: (i) indicate what is on the vertical axis (time, $t$, is already on the horizontal axis); (ii) draw the path correctly; and (iii) indicate $c_0^*$, $c_1^*$, and $\widehat{t}$ correctly on relevant axes. [2 marks]

**3. Growth Rates [4 marks]**

Let $P(t)$ be the value of an asset at time $t$, and let $G(t) = \int_0^t g(\tau)d\tau$, where $g(t)$ is the growth rate of $P(t)$ at time $t$, i.e., $g(t) = \dot{P}(t)/P(t)$.

**(a)** Find an expression for the value of the asset at time $T > 0$. That is, find $P(T)$. Your answer should be in terms of $G(T)$, $P(0)$, and the number $e$. [1 mark]

**(b)** Find an expression for $\ln[P(T)]$. Your answer should be in terms of $G(T)$ and $P(0)$, and may involve the logarithmic function, ln. [1 mark]

**(c)** Let $\overline{g}$ and $\underline{g}$ be constants which do not depend on time, $t$, and assume that $0 < \underline{g} < \overline{g}$. Let $g(t) = \underline{g}$ for all $t \in [0, T)$, and let $g(t) = \overline{g}$ for all $t \geq T$. Draw the time path of $\ln[P(t)]$. Indicate $T$ on suitable axis. [2 marks]

## Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for <u>Problem 1</u>

| Student Name: | SID Number: |
|---|---|

Answer to 1 (a):

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{\lambda}}$$

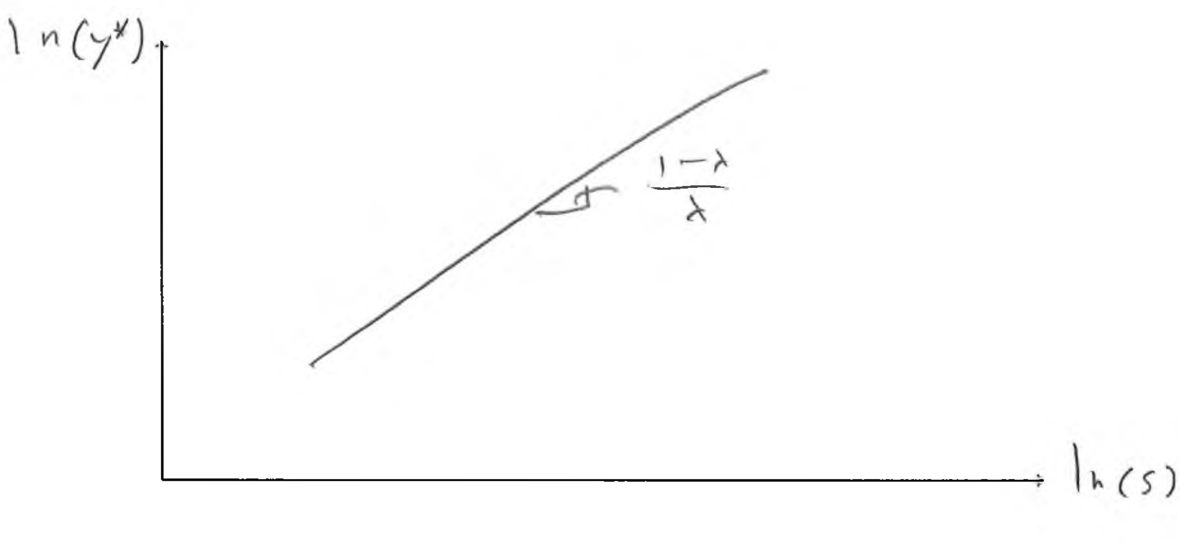
Answer to 1 (b):

$$\frac{k^*}{y^*} = \frac{s}{n+g+\delta}$$

Answer to 1 (c):

$$r^* = (1-\lambda)\left(\frac{n+g+\delta}{s}\right) - \delta$$

Answer to 1 (d):



**Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for Problem 2**

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|---|---|
| Student Name: | SID Number: |

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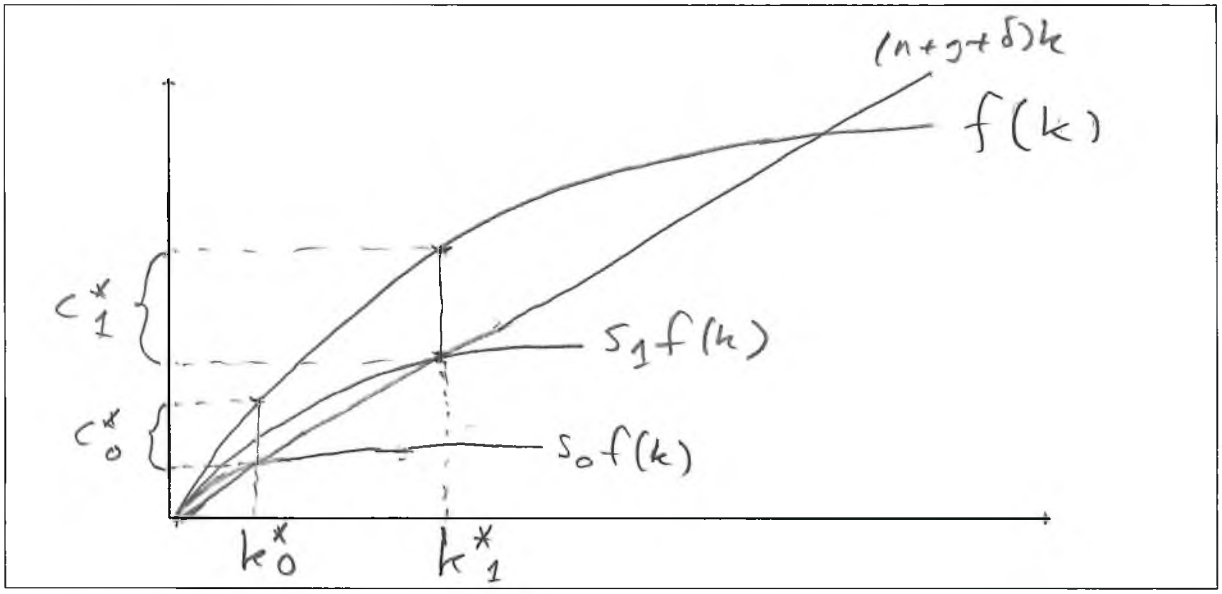
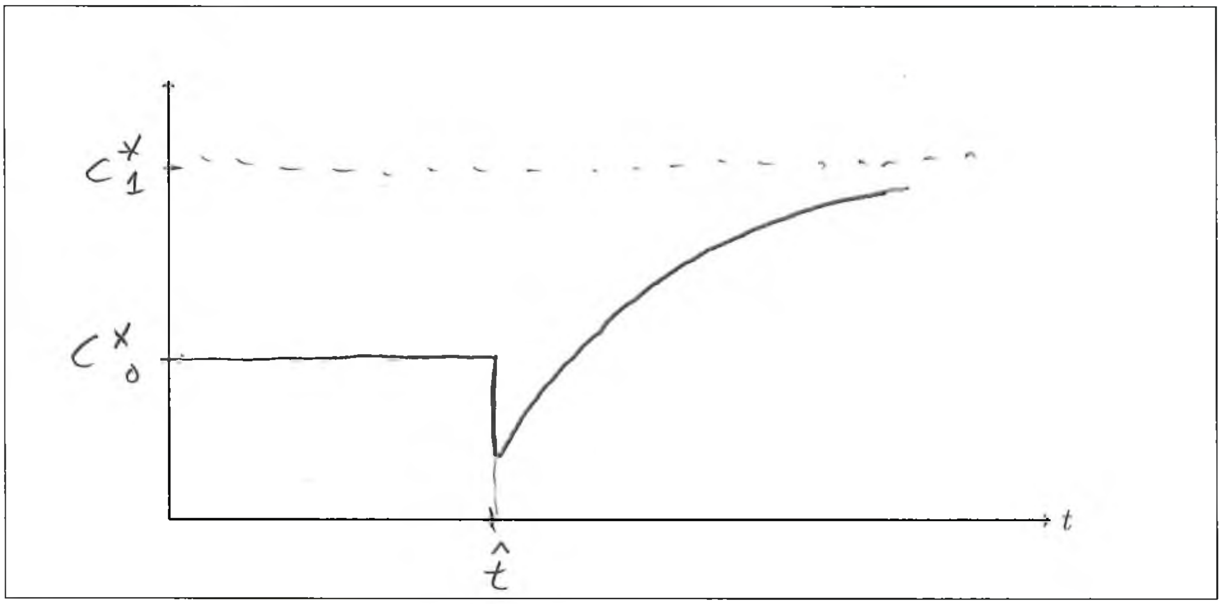
Answer to 2 (a):




Answer to 2 (b):



**Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for Problem 3**

| Student Name: | SID Number: |
|---|---|

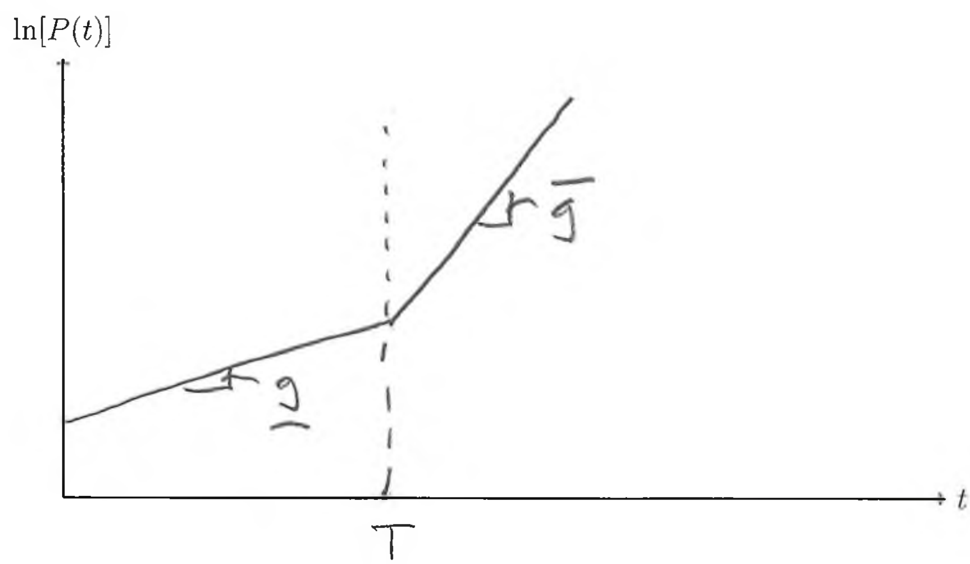
Answer to 3 (a):

$$P(T) = P(0)\, e^{g(T)}$$

Answer to 3 (b):

$$\ln[P(T)] = \ln[P(0)] + g(T)$$

Answer to 3 (c):



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**1. The Solow Model I** [**4 marks**]

Consider a Solow model with Cobb-Douglas production. The capital stock per effective worker, $k$, evolves over time according to

$$\dot{k} = sZk^{\alpha} - (n + g + \delta)k,$$

where the parameters $Z$, $\alpha$, $s$, $n$, $g$, and $\delta$ are strictly positive and constant, and $0 < \alpha < 1$ and $0 < s < 1$. Let $y = Zk^{\alpha}$ be output per effective worker, and let $k^*$ and $y^*$ be the steady-state levels of $k$ and $y$, respectively.

**(a)** Find $k^*$ in terms of (some or all of) $Z$, $\alpha$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(b)** Find $k^*/y^*$ in terms of (some or all of) $Z$, $\alpha$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(c)** Let $r = \frac{\partial y}{\partial k} - \delta$. Find an expression for the steady-state level of $r$, denoted $r^*$. Your answer should be in terms of (some or all of) $Z$, $\alpha$, $s$, $n$, $g$, and $\delta$. [1 mark]

**(d)** Let $c^* = (1 - s)y^*$ be steady-state consumption per effective worker. In the diagram provided, indicate $s$ on the horizontal axis and $c^*$ on the vertical axis and draw the graph of $c^*$. Also indicate $\alpha$ on suitable axis. [1 mark]

**2. The Solow Model II [4 marks]**

Consider a standard Solow model, where the capital stock per effective worker, $k$, evolves over time according to

$$\dot{k} = sf(k) - (n + g + \delta)k.$$

Here $f(k)$ is a general intensive-form production function, such that $f(0) = 0$, $f'(k) > 0$ and $f''(k) < 0$. As in class, we let $c = (1 - s)f(k)$ be consumption per effective worker, where $s$ is the rate of saving (or investment), such that $0 < s < 1$. The parameters $n$, $g$, and $\delta$ are strictly positive and constant.

Assume that the economy is initially in a steady state associated with $s = s_0$. Then $s$ changes from $s_0$ to $s_1$, from some point in time $\widehat{t}$, and then stays at $s_1$ forever. We also assume that

$$0 < s_{\text{GR}} < s_0 < s_1 < 1,$$

where $s_{\text{GR}}$ is the golden-rule level of $s$, which maximizes steady-state consumption per effective worker, $c^*$.

Let $k_0^*$ and $k_1^*$ be the steady-state levels of $k$ associated with $s_0$ and $s_1$, respectively. Similarly, let $c_0^*$ and $c_1^*$ be the steady-state levels of $c$ associated with $s_0$ and $s_1$, respectively.

**(a)** In the diagram provided, draw the time path for $k$. For full mark you must: (i) indicate time, $t$, on the horizontal axis; (ii) indicate $k$ on the vertical axis; (iii) draw the time path of $k$ correctly; (iv) indicate $k_0^*$, $k_1^*$, and $\widehat{t}$ on relevant axes; and (v) indicate the slope of the path at time $\widehat{t}$. [2 marks]

**(b)** In the diagram provided, draw the time path for $c$. For full mark you must: (i) indicate what is on the vertical axis (time, $t$, is already on the horizontal axis); (ii) draw the path correctly; and (iii) indicate $c_0^*$, $c_1^*$, and $\widehat{t}$ correctly on relevant axes. [2 marks]

**3. Growth Rates [4 marks]**

Let $P(t)$ be the value of an asset at time $t$, and let $R(t) = \int_0^t r(\tau)d\tau$, where $r(t)$ is the growth rate of $P(t)$ at time $t$, i.e., $r(t) = \dot{P}(t)/P(t)$.

**(a)** Find an expression for the value of the asset at time $T > 0$. That is, find $P(T)$. Your answer should be in terms of $R(T)$, $P(0)$, and the number $e$. [1 mark]

**(b)** Find an expression for $\ln[P(T)]$. Your answer should be in terms of $R(T)$ and $P(0)$, and may involve the logarithmic function, ln. [1 mark]

**(c)** Let $\overline{r}$ and $\underline{r}$ be constants which do not depend on time, $t$, and assume that $0 < \underline{r} < \overline{r}$. Let $r(t) = \overline{r}$ for all $t \in [0, T)$, and let $r(t) = \underline{r}$ for all $t \geq T$. Draw the time path of $\ln[P(t)]$. Indicate $T$ on suitable axis. [2 marks]

**Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for Problem 1**

Student Name:

SID Number:

Answer to 1 (a):

$$k^* = \left(\frac{sz}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

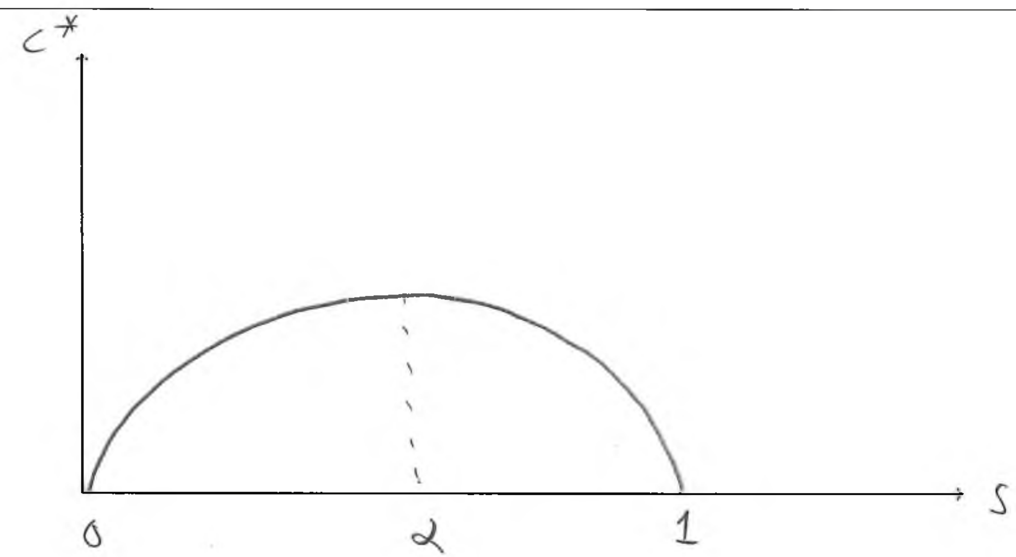
Answer to 1 (b):

$$\frac{k^*}{y^*} = \frac{s}{n+g+\delta}$$

Answer to 1 (c):

$$r^* = \alpha\left(\frac{n+g+\delta}{s}\right) - \delta$$

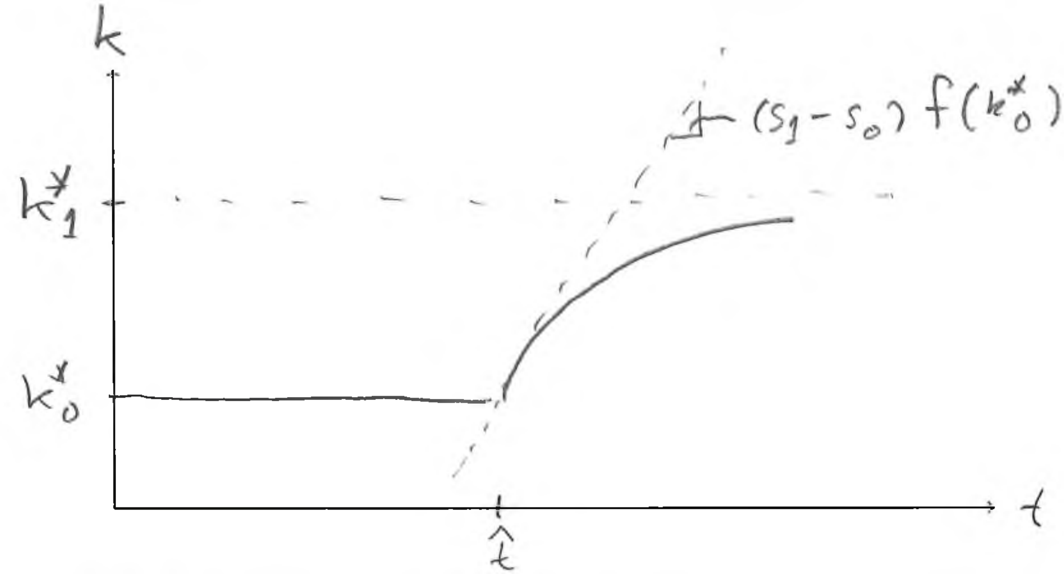
Answer to 1 (d):



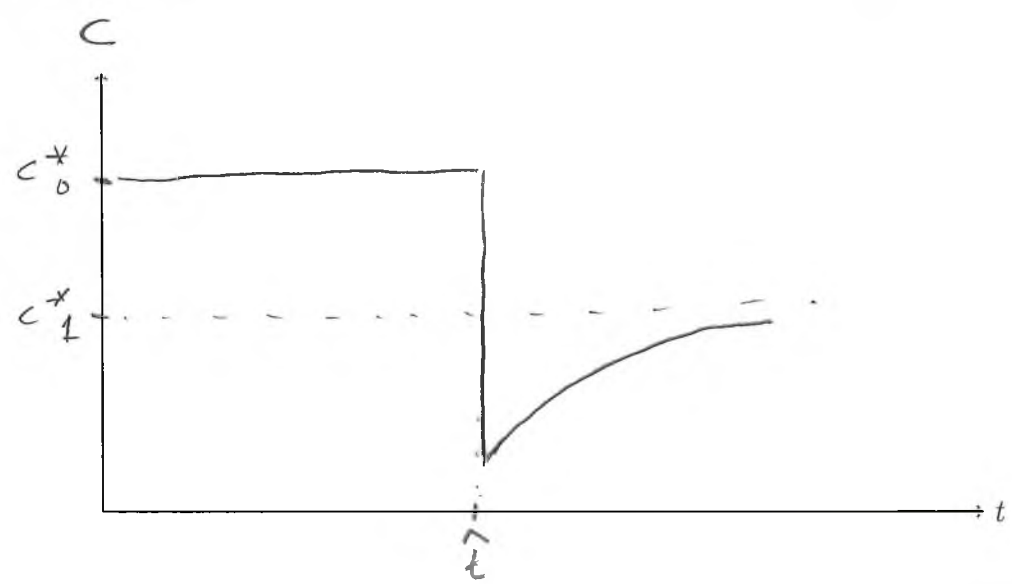
Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for Problem 2

Student Name: SID Number:

Answer to 2 (a):




Answer to 2 (b):



Econ 4020 Midterm Exam February 10, 2025 – Answer sheet for Problem 3

Student Name: SID Number:

Answer to 3 (a):

$$P(T) = P(0)e^{R(T)}$$

Answer to 3 (b):

$$\ln[P(T)] = \ln[P(0)] + R(T)$$

Answer to 3 (c):