

Some practice problems for Econ 4020

Problem 1: Let $f(x) = 2x$. Find the following:

- (a) $f(1)$
- (b) $f(2)$
- (c) $f(1/2)$
- (d) $f(a)$ (the answer should involve a and/or numbers)

Problem 2: Let $f(x) = x^2/2$. Find the following:

- (a) $f(1)$
- (b) $f(2)$
- (c) $f(1/2)$
- (d) $f(a)$ (the answer should involve a and/or numbers)

Problem 3: Let $f(x) = \ln(x)$. Find the following:

- (a) $f(1)$
- (b) $f(2)$
- (c) $f(1/2)$
- (d) $f(a)$ (the answer should involve a and/or numbers)

Problem 4: Let $f(x) = 4^x$. Find the following:

- (a) $f(1)$
- (b) $f(2)$
- (c) $f(1/2)$
- (d) $f(a)$ (the answer should involve a and/or numbers)

Problem 5: Let $f(x) = e^x$. Find the following:

- (a) $f(1)$
- (b) $f(2)$
- (c) $f(1/2)$
- (d) $f(a)$ (the answer should involve a and/or numbers)

Problem 6: Find the first derivatives of the following functions, where a and b are strictly positive constants:

- (a) $f(x) = x^2/2$
- (b) $f(x) = ax - bx^2$
- (c) $f(x) = ae^{bx}$
- (d) $f(x) = a \ln(x + b)$

Problem 7: Find expressions for $f'(1)$ for the following functions, where a and b are strictly positive constants:

(a) $f(x) = x^2/2$

(b) $f(x) = ax - bx^2$

(c) $f(x) = ae^{bx}$

(d) $f(x) = a \ln(x + b)$

Problem 8: Find expressions for $f'(0)$ for the following functions, where a and b are strictly positive constants:

(a) $f(x) = x^2/2$

(b) $f(x) = ax - bx^2$

(c) $f(x) = ae^{bx}$

(d) $f(x) = a \ln(x + b)$

Problem 9: Let x^* be the level of x that maximizes $f(x)$. Use the first-order condition to find x^* for the following examples of $f(x)$, where a and b are strictly positive constants:

(a) $f(x) = x - x^2/2$

(b) $f(x) = ax - bx^2$

(c) $f(x) = a \ln(x) - bx$

(d) $f(x) = x^a - bx$ (here we also assume that $0 < a < 1$)

Problem 10: Draw the graphs of the following functions for $x \geq 0$. Indicate vertical intercepts (if any) and other points on the axes that help explain the shape of the graphs. Both a and b are strictly positive constants.

(a) $f(x) = 1 + x$

(b) $f(x) = 1 + 2x$

(c) $f(x) = 2 + x/2$

(d) $f(x) = a + bx$

(e) $f(x) = a + bx^2$

(f) $f(x) = a - bx^2$

(g) $f(x) = a + b/x$ (here we also assume $x > 0$)

(h) $f(x) = a + b/(x + 1)$

(i) $f(x) = a + b/(x - 1)$ (here we also assume $x > 1$)

Solutions:

Problem 1: (a) 2; (b) 4; (c) 1; (d) $2a$

Problem 2: (a) $1/2$; (b) 2; (c) $1/8$; (d) $a^2/2$

Problem 3: (a) 0; (b) $\ln(2)$; (c) $\ln(1/2) = -\ln(2)$; (d) $\ln(a)$

Problem 4: (a) 4; (b) 16; (c) 2; (d) 4^a

Problem 5: (a) e ; (b) e^2 ; (c) $e^{1/2} = \sqrt{e}$; (d) e^a

Problem 6: (a) $f'(x) = x$; (b) $f'(x) = a - 2bx$; (c) $f'(x) = abe^{bx}$; (d) $f'(x) = a/(b + x)$

Problem 7: (a) $f'(1) = 1$; (b) $f'(1) = a - 2b$; (c) $f'(1) = abe^b$; (d) $f'(1) = a/(b + 1)$

Problem 8: (a) $f'(0) = 0$; (b) $f'(0) = a$; (c) $f'(0) = ab$; (d) $f'(0) = a/b$

Problem 9: (a) $x^* = 1$; (b) $x^* = a/(2b)$; (c) $x^* = a/b$; $x^* = \left(\frac{a}{b}\right)^{\frac{1}{1-a}}$

Problem 10: See figures

Solutions to Problem 10

















